MA99 EDUCATION

Mathematics Extension 1: Practice Paper 1

01

NSW MATHEMATICS EXTENSION 1 11-12 SYLLABUS 2024

FOR HSC 2028 AND LATER

General Instructions

- Reading time: 10 minutes
- Working time: 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- The NESA Mathematics Extension 1 reference sheet may be used.
- In Questions 11 onwards, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I · 10 marks

Questions 1 to 10

Suggested time: about 15 minutes

Section II · 60 marks

Questions 11 to 14

Suggested time: about 1 hour 30 minutes (a minute and a half per mark), which leaves about 15 minutes to check your work

Year 11 Mathematics Advanced is assumed knowledge for this paper.

This paper examines the whole Year 11 Mathematics Extension 1 course of the 2024 syllabus: further work with functions, polynomials, further trigonometry, permutations and combinations, and the binomial theorem. Questions with a setting are drawn from engineering, design and construction.

Marks may be deducted for careless or badly arranged work.

Section I · Answer Sheet

Shade ONE circle for each of Questions 1 to 10. If you change your mind, draw a cross through the incorrect answer and shade the new one.

1	A	B	C	D	2	A	B	C	D
3	A	B	C	D	4	A	B	C	D
5	A	B	C	D	6	A	B	C	D
7	A	B	C	D	8	A	B	C	D
9	A	B	C	D	10	A	B	C	D

Section I · 10 marks

Answer Questions 1 to 10 on the multiple choice answer sheet, spending about 15 minutes on this section.

Question 1

1 MARK

When $P(x) = 2x^3 + x^2 + kx - 4$ is divided by $x + 2$, the remainder is -6. What is the value of k?

A -11
B -8
C -5
D 5

Question 2

1 MARK

The tip of a robotic welding arm moves so that its position, in centimetres, is given by $x = 2 + 3\cos t$ and $y = -1 + 3\sin t$, for $0 \leq t \leq 2\pi$Which Cartesian equation describes the path of the tip?

A $(x-2)^2 + (y+1)^2 = 9$
B $(x+2)^2 + (y-1)^2 = 9$
C $(x-2)^2 + (y+1)^2 = 3$
D $(x-2)^2 + (y+1)^2 = 1$

Question 3

1 MARK

What is the coefficient of x^3 in the expansion of $(2 - x)^7$?

A 560
B -280
C -35
D -560

Question 4

1 MARK

Which of the following is the solution of $|3 - 2x| > 5$?

A $-1 < x < 4$
B $x < -1$ or $x > 4$
C $x < -4$ or $x > 1$
D $x < -1$

Question 5

1 MARK

The letters of the word CONCRETE are arranged in a row. In how many of the arrangements are the two letters C next to each other?

A 2520
B 1260
C 5040
D $10\,080$

Question 6

1 MARK

The expression $\cos x - \sqrt{3}\sin x$ is written in the form $R\cos(x + \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. Which of the following is correct?

A $2\cos\left(x - \frac{\pi}{3}\right)$
B $4\cos\left(x + \frac{\pi}{3}\right)$
C $2\cos\left(x + \frac{\pi}{3}\right)$
D $2\cos\left(x + \frac{\pi}{6}\right)$

Question 7

1 MARK

Which of the following is equal to ${}^{12}C_5 + {}^{12}C_6$?

A ${}^{13}C_5$
B ${}^{13}C_6$
C ${}^{24}C_{11}$
D ${}^{12}C_{11}$

Question 8

1 MARK

Eight people, including the site manager and the architect, sit at a round table with eight equally spaced seats for a project meeting. Only their positions relative to one another matter. In how many ways can they sit if the site manager and the architect sit directly opposite each other?

A 1440
B 5040
C 5760
D 720

Question 9

1 MARK

The angles A and B are acute, with $\tan A = 2$ and $\tan B = 3$. What is the value of $A + B$?

A 45°
B 135°
C -45°
D 315°

Question 10

1 MARK

The equation $2x^3 - 3x^2 - 4x + 5 = 0$ has roots α, β and γ. What is the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$?

A $\frac{4}{5}$
B $-\frac{4}{5}$
C $-\frac{3}{5}$
D $\frac{5}{4}$

Section II · 60 marks

Answer Questions 11 to 14 in the spaces provided, spending about 1 hour 30 minutes on this section. Show your mathematical reasoning and/or calculations in every answer.

Question 11 · 15 marks

(a) Find the term independent of x in the expansion of $\left(x^3 - \dfrac{2}{x}\right)^8$.

2 MARKS

(b) Let $P(x) = x^3 - 2x^2 - 5x + 6$.

(i) Show that $x - 1$ is a factor of $P(x)$. 1 MARK

(ii) Hence, factorise $P(x)$ fully. 2 MARKS

(iii) Hence, solve $P(x) < 0$. 2 MARKS

(c) A designer mounts eight different panels in a row along a building facade. Five of the panels are glass and three are timber.

(i) How many different arrangements of the eight panels are possible? 1 MARK

(ii) In how many of these arrangements are the three timber panels next to each other? 2 MARKS

(d) Show that $\sin(x + 30^\circ) + \cos(x + 60^\circ) = \cos x$. 2 MARKS

(e) Solve $\cos 2x = \cos x$ for $0 \leq x \leq 2\pi$. 3 MARKS

Question 12 · 15 marks

(a) Find the coefficient of x^2 in the expansion of $(2 + x)^5(1 - 3x)^2$. 2 MARKS

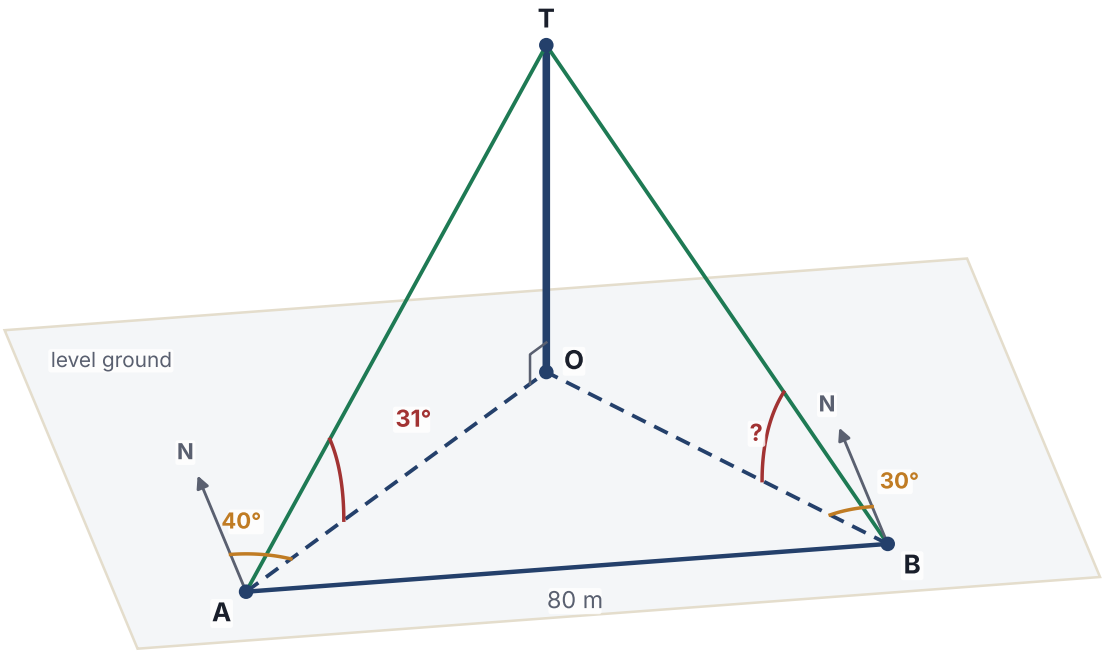
(b) Solve $\dfrac{2x + 1}{x - 3} \leq 1$. 3 MARKS

(c) The gate of a construction site opens with a four digit access code. The digits are chosen from $0, 1, 2, \ldots, 9$, no digit may be repeated, and a code may begin with 0.

(i) How many different access codes are possible? 1 MARK

(ii) A code is chosen at random. What is the probability that its digits are in increasing order from left to right? 2 MARKS

(d) A vertical crane mast OT stands with its foot O on level ground. Points A and B are on the same level ground, with B 80 m due east of A. From A, the bearing of O is 040° and the angle of elevation of T is 31°. From B, the bearing of O is 330°. Find, to the nearest degree, the angle of elevation of T from B. 4 MARKS

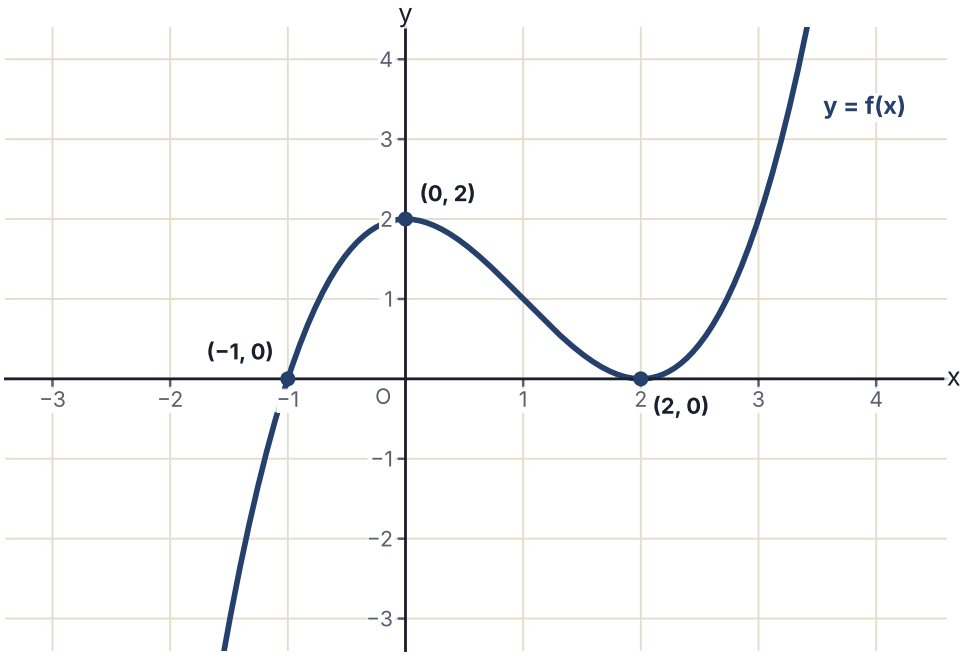


Question 12 (d) · the crane mast OT (not to scale)

(e) The length, width and depth, in metres, of a rectangular concrete footing are the three roots of $x^3 - 9x^2 + kx - 15 = 0$, where k is a constant. The three dimensions can be written as $a - d$, a and $a + d$, where $a > d > 0$. Use the sums and products of the roots to find the dimensions of the footing and the value of k. 3 MARKS

Question 13 · 15 marks

(a) The graph of $y = f(x)$, where $f(x) = \frac{1}{2}(x+1)(x-2)^2$, is shown. It has a local maximum at $(0, 2)$ and touches the x-axis at $(2, 0)$. On the diagram, sketch the graph of $y = \frac{1}{f(x)}$, showing all asymptotes and the coordinates of any turning point. 3 MARKS



Question 13 (a) · the graph of $y = f(x)$

(b) A batch of 12 bolts for a steel frame contains 3 defective bolts. An inspector selects 4 bolts from the batch at random. Find the probability that at least one of the selected bolts is defective. 2 MARKS

(c) Let $f(x) = 1 + \sqrt{x+3}$, for $x \geq -3$.

(i) Explain why f has an inverse function, and find $f^{-1}(x)$, stating its domain. 3 MARKS

(ii) Find the exact coordinates of any point where the graphs of $y = f(x)$ and $y = f^{-1}(x)$ intersect. 2 MARKS

(d) The load L, in kilonewtons, carried by a bracing strut that makes an angle θ with the horizontal is modelled by $L = 12\cos\theta + 5\sin\theta$, for $0^\circ \leq \theta \leq 90^\circ$.

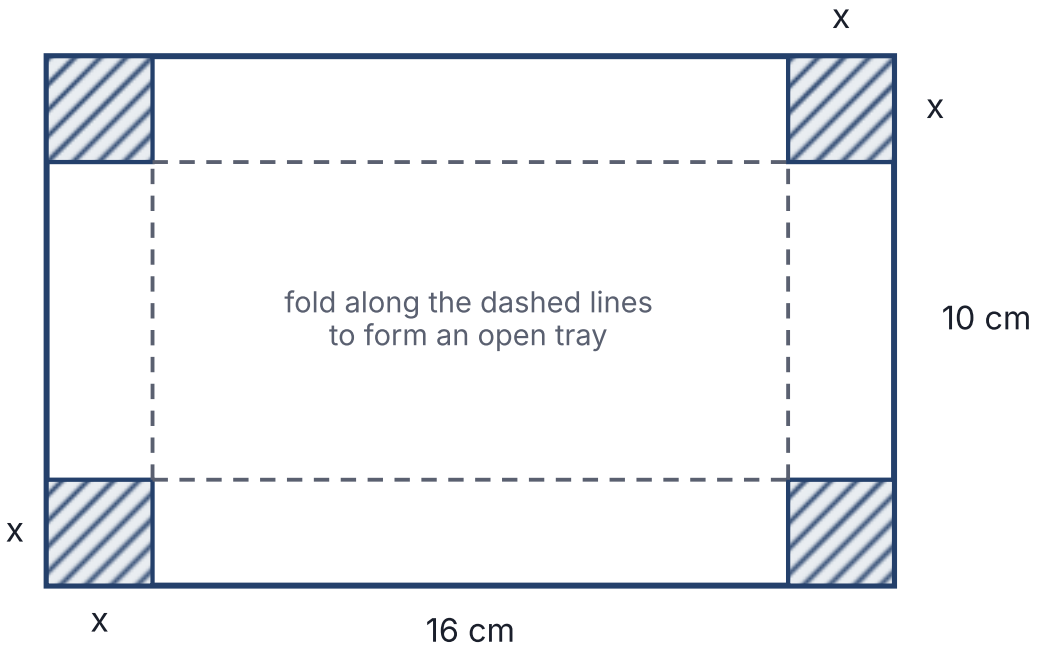
(i) Express $12\cos\theta + 5\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$ with α correct to the nearest minute. 2 MARKS

(ii) The strut is safe when the load is at most 10 kilonewtons. Find the values of θ for which the strut is safe, giving the boundary angle correct to the nearest minute. 3 MARKS

Question 14 · 15 marks

(a) A design panel of 4 people is to be chosen from 4 engineers, 3 architects and 2 planners. In how many ways can the panel be chosen if it must include at least one person from each profession? 3 MARKS

(b) An open tray is made from a rectangular metal sheet 16 cm by 10 cm. A square of side x cm is cut from each corner and the sides are folded up, as shown.



Question 14 (b) · the sheet before folding

(i) Show that the volume of the tray, V cm^3, is given by $V = 4x^3 - 52x^2 + 160x$. 1 MARK

(ii) Show that $V - 144 = 4(x-2)^2(x-9)$. 2 MARKS

(iii) Hence explain why no tray made in this way can hold more than 144 cm^3, and find the dimensions of the tray that holds exactly 144 cm^3. 3 MARKS

(c) Let n be a positive integer.

(i) Write down the coefficient of x^{2k} in the expansion of $\left(1-x^2\right)^n$, where k is an integer with $0 \leq k \leq n$. 1 MARK

(ii) By considering $\left(1-x^2\right)^n = (1-x)^n(1+x)^n$, show that when $n = 2m$ is even, 3 MARKS

$$\sum_{r=0}^{2m} (-1)^r \left({}^{2m}C_r\right)^2 = (-1)^m \, {}^{2m}C_m.$$

(iii) Explain why $\sum_{r=0}^{n} (-1)^r \left({}^{n}C_r\right)^2 = 0$ when n is odd. 2 MARKS

End of paper.

About this resource

MA99 EDUCATION

Scope

NSW Mathematics Extension 1, Year 11. Written to the NSW Mathematics Extension 1 11-12 Syllabus (2024), all Year 11 focus areas, for students sitting the HSC in 2028 and later years. Every value and solution was re-derived independently by computer algebra before printing. Contexts in the questions are invented unless a source is named.

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MA99 EDUCATION

Mathematics Extension 1: Practice Paper 1 · Worked Solutions

01

NSW MATHEMATICS EXTENSION 1 11-12 SYLLABUS 2024 FOR HSC 2028 AND LATER

Worked solutions · Section I

1

By the remainder theorem the remainder on division by $x+2$ is $P(-2)$:

$$P(-2) = 2(-8) + 4 - 2k - 4 = -16 - 2k$$

$$-16 - 2k = -6 \implies -2k = 10 \implies k = -5$$

(C)

(A) -11 comes from substituting $x = 2$ instead of $x = -2$: $16 + 2k = -6$. (B) -8 comes from setting $P(-2) = 0$ (treating $x+2$ as a factor) rather than using the remainder -6. (D) 5 is a sign slip in the last step, dividing 10 by 2 instead of by -2.

2

$\cos t = \dfrac{x-2}{3}$ and $\sin t = \dfrac{y+1}{3}$, and $\cos^2 t + \sin^2 t = 1$, so $\dfrac{(x-2)^2}{9} + \dfrac{(y+1)^2}{9} = 1$, that is $(x-2)^2 + (y+1)^2 = 9$: a circle with centre $(2, -1)$ and radius 3, traced once.

(A)

(B) reverses the signs of the centre, reading $(-2, 1)$ from the equations. (C) uses the radius 3 without squaring it. (D) writes $\cos t = x - 2$ and $\sin t = y + 1$, forgetting to divide by 3.

3

The general term of $(2-x)^7$ is ${}^7C_r\, 2^{7-r}(-x)^r$. For x^3, $r = 3$:

$$ {}^7C_3\, 2^4(-1)^3 = 35 \times 16 \times (-1) = -560$$

(D)

(A) loses the sign of $(-x)^3$. (B) -280 uses 2^3 instead of $2^{7-3} = 2^4$, attaching the power r to the wrong term. (C) -35 is ${}^7C_3$ with the sign but without the power of 2.

4

$|3 - 2x| > 5$ means $3 - 2x > 5$ or $3 - 2x < -5$. The first gives $-2x > 2$, so $x < -1$; the second gives $-2x < -8$, so $x > 4$.

(B)

(A) is the solution of $|3 - 2x| < 5$: the distance from the centre is greater than 5, so the solution lies outside the interval, not inside. (C) is the solution of $|2x + 3| > 5$: a sign slip when $|3 - 2x|$ is rewritten as $|2x + 3|$ instead of $|2x - 3|$. (D) writes $3 - 2x > \pm 5$, the error NESA warns about: $3 - 2x > -5$ gives $x < 4$, and combining it with $x < -1$ loses the branch $x > 4$.

5

Glue the two Cs into one block. The block and O, N, R, E, T, E are 7 objects with E repeated twice, and the block CC reads the

same either way round, so there are $\frac{}{2!} = 2520$ arrangements.

(A)

(B) $1260 = \dfrac{7!}{2!\,2!}$ still divides by $2!$ for the Cs, but the Cs are now one block and there is nothing left to over count. (C) $5040 = 7!$ forgets the repeated E. (D) $10\,080 = \dfrac{8!}{2!\,2!}$ counts every arrangement and ignores the condition.

6

$R\cos(x+\alpha) = R\cos x\cos\alpha - R\sin x\sin\alpha$ Matching with $\cos x - \sqrt{3}\sin x$: $R\cos\alpha = 1$ and $R\sin\alpha = \sqrt{3}$, so $R = \sqrt{1+3} = 2$ and $\tan\alpha = \sqrt{3}$, $\alpha = \dfrac{\pi}{3}$.

(C)

(A) expands $\cos(x+\alpha)$ with a plus sign in the middle, which flips the sign of α. (B) takes $R = 1^2 + (\sqrt{3})^2 = 4$ and forgets the square root. (D) uses $\tan\alpha = \dfrac{1}{\sqrt{3}}$, the ratio upside down.

7

Pascal's identity ${}^{n}C_{r} = {}^{n-1}C_{r-1} + {}^{n-1}C_{r}$ with $n = 13$, $r = 6$ gives ${}^{12}C_{5} + {}^{12}C_{6} = {}^{13}C_{6} = 1716$.

(B)

(A) keeps the smaller lower index: ${}^{13}C_{5} = 1287$. (C) adds both upper and both lower indices. (D) adds only the lower indices: ${}^{12}C_{11} = 12$.

8

Seat the site manager first; on a round table every seat is the same, so this is 1 way. The architect must take the seat opposite: 1 way. The other 6 people fill the remaining 6 seats, which are now distinguishable: $6! = 720$ ways.

(D)

(A) 1440 multiplies by 2 for swapping the manager and the architect, but once the manager is seated that swap is just a rotation. (B) $5040 = 7!$ ignores the condition. (C) $5760 = 8 \times 6!$ lets the manager choose any of 8 seats, counting each arrangement once for every rotation.

9

$$\tan(A+B) = \frac{2+3}{1-2\times 3} = \frac{5}{-5} = -1$$

Both tangents exceed 1, so $45^\circ < A, B < 90^\circ$ and $90^\circ < A+B < 180^\circ$. The angle in that interval with tangent -1 is 135°.

(B)

(A) writes the denominator as $\tan A\tan B - 1 = 5$, giving $\tan(A+B) = 1$. (C) is the calculator value $\tan^{-1}(-1)$, which is negative and cannot be a sum of two acute angles. (D) adds 360° to the calculator value; it has tangent -1 but is far larger than 180°.

10

$\Sigma\alpha\beta = \dfrac{c}{a} = \dfrac{-4}{2} = -2$ and $\alpha\beta\gamma = -\dfrac{d}{a} = -\dfrac{5}{2}$.

$$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma} = \frac{\beta\gamma+\gamma\alpha+\alpha\beta}{\alpha\beta\gamma} = \frac{-2}{-\frac{5}{2}} = \frac{4}{5}$$

(A)

(B) takes $\alpha\beta\gamma = +\dfrac{5}{2}$, forgetting the minus sign in $-\dfrac{d}{a}$. (C) puts the sum of the roots, $\dfrac{3}{2}$, on top instead of $\Sigma\alpha\beta$. (D) inverts the fraction.

Section I key

1 C · 2 A · 3 D · 4 B · 5 A · 6 C · 7 B · 8 D · 9 B · 10 A

For each item you missed, find the distractor you chose above: it names the exact error that produces it. Write that error down before you look at Section II.

Worked solutions · Section II

Question 11

11 (a)

$$T_{r+1} = {}^8C_r\,(x^3)^{8-r}\left(-\frac{2}{x}\right)^r = {}^8C_r(-2)^r x^{24-4r} \quad \text{general term}$$

$$24 - 4r = 0 \implies r = 6 \quad \text{independent of } x$$

$$T_7 = {}^8C_6(-2)^6 = 28 \times 64 = 1792$$

$\boxed{1792}$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Finds the general term and $r = 6$, or equivalent merit	1

$(-2)^6$ is positive. Students who carry the minus sign outside the power lose the final mark.

11 (b) (i)

$P(1) = 1 - 2 - 5 + 6 = 0$, so by the factor theorem $\boxed{x - 1 \text{ is a factor of } P(x)}$.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	1

11 (b) (ii)

Dividing $P(x)$ by $x - 1$ (or comparing coefficients) gives the quotient $x^2 - x - 6$:

$$P(x) = (x-1)(x^2 - x - 6) = (x-1)(x-3)(x+2)$$

$\boxed{P(x) = (x-1)(x-3)(x+2)}$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Finds the quadratic factor $x^2 - x - 6$, or equivalent merit	1

11 (b) (iii)

The zeroes are -2, 1 and 3, each single, and the leading coefficient is positive, so the cubic is negative to the left of -2, positive between -2 and 1, negative between 1 and 3 and positive after 3.

$x < -2$ or $1 < x < 3$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Correctly uses the zeroes and the sign of the cubic to obtain one correct interval, or equivalent merit	1

A quick sign diagram, or a rough sketch of $y = P(x)$ through the three zeroes, avoids the common error of giving $-2 < x < 1$, the region where the cubic is positive.

11 (c) (i)

Eight different panels in a row: $8! = 40\,320$ arrangements.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct answer	1

11 (c) (ii)

Treat the three timber panels as one block. The block and the five glass panels are 6 objects: $6!$ orders. Inside the block the timber panels can be ordered in $3!$ ways.

$$6! \times 3! = 720 \times 6 = 4320$$

4320

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Obtains $6!$ for the block and the glass panels, or equivalent merit	1

Forgetting the $3!$ inside the block gives 720. The panels are different, so their order inside the block matters.

11 (d)

$\text{LHS} = \sin x \cos 30^\circ + \cos x \sin 30^\circ + \cos x \cos 60^\circ - \sin x \sin 60^\circ$ both expansions

$= \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x + \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$ exact values

$= \cos x = \text{RHS}$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct proof	2
Correctly expands both compound angles, or equivalent merit	1

The minus sign in $\cos(A+B) = \cos A \cos B - \sin A \sin B$ is the step that makes the sine terms cancel. Write the expansions in full before substituting the exact values.

11 (e)

$2\cos^2 x - 1 = \cos x$ $\cos 2x = 2\cos^2 x - 1$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$ factorise

$\cos x = 1 : \ x = 0,\ 2\pi$

$\cos x = -\frac{1}{2} : \ x = \frac{2\pi}{3},\ \frac{4\pi}{3}$ second and third quadrants

$x = 0, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, 2\pi$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Obtains $(2\cos x + 1)(\cos x - 1) = 0$ and finds the solutions of one factor, or equivalent merit	2
Uses $\cos 2x = 2\cos^2 x - 1$ to obtain a quadratic in $\cos x$, or equivalent merit	1

The domain is closed, so both endpoints 0 and 2π are solutions. Missing 2π is the most common way to lose the last mark.

Question 12

12 (a)

$(2+x)^5 = 32 + 80x + 80x^2 + \cdots$ and $(1-3x)^2 = 1 - 6x + 9x^2$. The x^2 terms of the product come from three pairs:

$$80 \times 1 + 80 \times (-6) + 32 \times 9 = 80 - 480 + 288 = -112$$

-112

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Finds the first three terms of $(2+x)^5$, or identifies the three products that give x^2, or equivalent merit	1

Only three products give x^2. Students who expand everything waste time and usually make an arithmetic slip.

12 (b)

$x \neq 3$. Multiply both sides by $(x-3)^2$, which is positive:

$$(2x+1)(x-3) \leq (x-3)^2$$

$$(x-3)\left[(2x+1)-(x-3)\right] \leq 0 \quad \text{collect on one side and factorise}$$

$$(x-3)(x+4) \leq 0$$

$$-4 \leq x \leq 3, \quad x \neq 3$$

$-4 \leq x < 3$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Obtains $(x-3)(x+4) \leq 0$, or equivalent merit	2
Multiplies by $(x-3)^2$, or finds the critical values -4 and 3, or equivalent merit	1

Multiplying both sides by $x-3$ without considering its sign gives $2x+1 \leq x-3$ so the wrong answer $x \leq -4$: for $x<3$ the factor $x-3$ is negative and would reverse the inequality. Multiplying by $(x-3)^2$ avoids the cases. And $x=3$ must be excluded: the expression is undefined there.

12 (c) (i)

Ordered selections of 4 digits from 10: ${}^{10}P_4 = 10 \times 9 \times 8 \times 7 = 5040$.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct answer	1

12 (c) (ii)

Each set of 4 different digits can be written in increasing order in exactly one way, so the number of increasing codes is the number of sets, ${}^{10}C_4 = 210$.

$$P(\text{increasing}) = \frac{210}{5040} = \frac{1}{24}$$

$\frac{1}{24}$

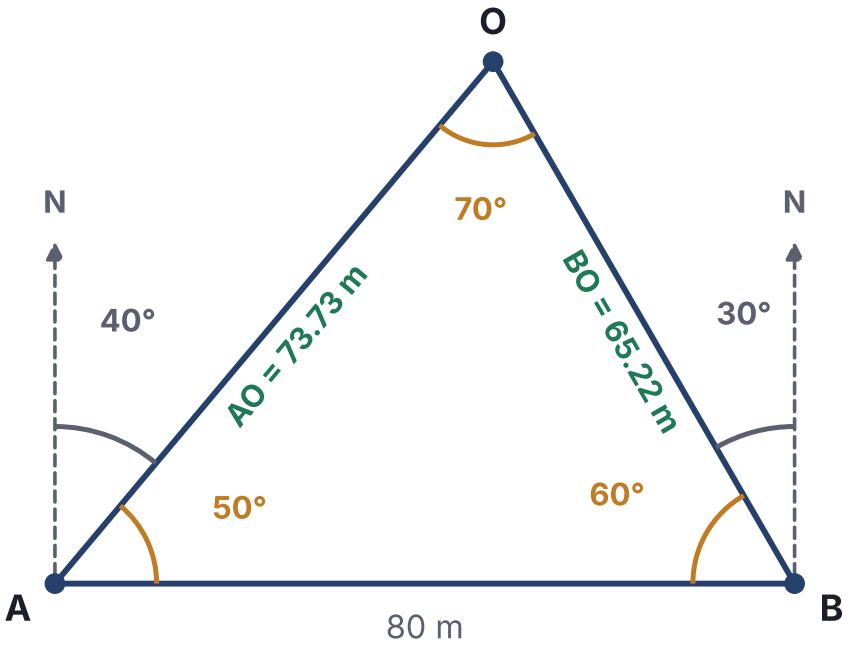
MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Recognises that there are ${}^{10}C_4$ increasing codes, or equivalent merit	1

Equivalently, the $4!$ orders of any chosen set are equally likely and exactly one is increasing, so the probability is $\frac{1}{4!}$.

12 (d)

In triangle AOB on the ground: AB points due east, so $\angle OAB = 90^\circ - 40^\circ = 50^\circ$ and $\angle OBA = 330^\circ - 270^\circ = 60^\circ$. Hence $\angle AOB = 70^\circ$.



Question 12 (d) · the plan of triangle AOB on the ground

$$AO = \frac{80 \sin 60^\circ}{\sin 70^\circ} \approx 73.73 \text{ m}, \qquad BO = \frac{80 \sin 50^\circ}{\sin 70^\circ} \approx 65.22 \text{ m} \quad \text{sine rule}$$

$$h = AO \tan 31^\circ \approx 44.30 \text{ m} \quad \text{right triangle } AOT$$

$$\tan \phi = \frac{h}{BO} \approx \frac{44.30}{65.22} \approx 0.6793 \quad \text{right triangle } BOT$$

$$\phi \approx 34^\circ 11'$$

The angle of elevation of T from B is $\boxed{34^\circ}$ (to the nearest degree).

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	4
Finds h and BO, or equivalent merit	3
Finds AO or BO using the sine rule, or equivalent merit	2
Finds the angles of triangle AOB, or equivalent merit	1

Draw the ground triangle flat first, with the north lines, and mark the bearings on it. The angles at A and B are measured from AB, not from north, which is where most errors start.

12 (e)

$(a-d)+a+(a+d)=3a=9 \implies a=3 \quad \Sigma\alpha=-\frac{b}{a}$

$(a-d)\,a\,(a+d)=3(9-d^2)=15 \implies d^2=4,\ d=2 \quad \alpha\beta\gamma=-\frac{d}{a}, d>0$

$\text{roots } 1,\ 3,\ 5$

$k=1\times3+3\times5+5\times1=23 \quad \Sigma\alpha\beta=\frac{c}{a}$

The footing is 1 m by 3 m by 5 m and $k=23$.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Finds $a=3$ and $d=2$, or equivalent merit	2
Uses the sum of the roots to find $a=3$, or equivalent merit	1

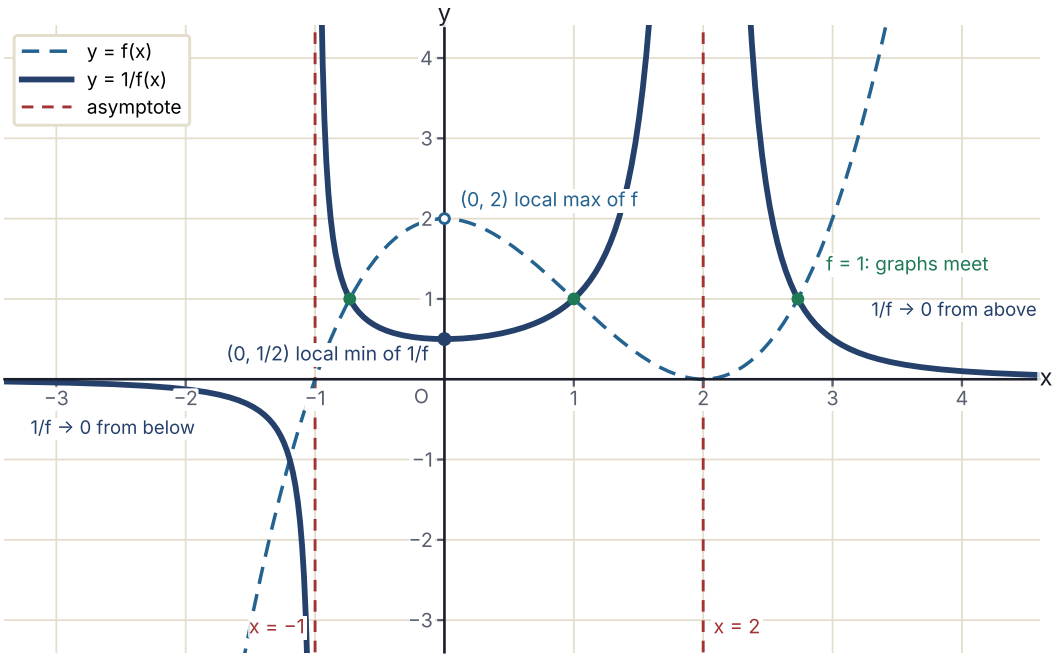
Here the letter d is the common difference, not the constant term of the cubic. The product of the roots is $-\frac{-15}{1}=15$, the volume of the footing in cubic metres.

Question 13

13 (a)

Read the critical points off $y=f(x)$ first.

- Zeroes of f at $x=-1$ and $x=2$ become vertical asymptotes of $y=\frac{1}{f(x)}$. At $x=2$, f touches the axis and stays positive, so $\frac{1}{f}\to+\infty$ on **both** sides.
- For $x<-1$, $f<0$ and $f\to-\infty$as $x\to-\infty$ so $\frac{1}{f}<0$ and $\frac{1}{f}\to0^-$: the x-axis is a horizontal asymptote. For $x>2$, $\frac{1}{f}\to0^+$.
- The local maximum $(0,2)$ of f becomes a local minimum $\left(0,\frac{1}{2}\right)$ of $\frac{1}{f}$.
- Where $f=1$ the two graphs meet (at $x=1$ and $x=1\pm\sqrt{3}$).



Question 13 (a) · $y = f(x)$ (dashed) and $y = \dfrac{1}{f(x)}$ (solid)

Asymptotes $x = -1$, $x = 2$ and $y = 0$; local minimum $\left(0, \frac{1}{2}\right)$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct sketch	3
Correct sketch with one error, for example the branch for $x < -1$ or the behaviour near $x = 2$, or equivalent merit	2
Shows the vertical asymptotes $x = -1$ and $x = 2$, or the local minimum $\left(0, \frac{1}{2}\right)$, or equivalent merit	1

The double zero at $x = 2$ is the trap. Because f does not change sign there, both branches of $\frac{1}{f}$ go up to $+\infty$; drawing one branch down to $-\infty$loses a mark.

13 (b)

Count the complement: no defective bolts means all 4 come from the 9 good bolts.

$$P(\text{none defective}) = \frac{{}^{9}C_{4}}{{}^{12}C_{4}} = \frac{126}{495} = \frac{14}{55}$$

$$P(\text{at least one defective}) = 1 - \frac{14}{55} = \frac{41}{55}$$

$\dfrac{41}{55}$ (about 0.745)

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Finds $\dfrac{{}^{9}C_{4}}{{}^{12}C_{4}}$, or equivalent merit	1

Counting directly, ${}^{3}C_{1} \times {}^{11}C_{3}$ is wrong: it counts a selection with two defective bolts more than once. The complement avoids the cases.

13 (c) (i)

$\sqrt{x+3}$ increases as x increases, so f is increasing on its domain $x \geq -3$ Every horizontal line meets $y = f(x)$ at most once, so f is one-to-one and has an inverse function.

The range of f is $y \geq 1$(as $\sqrt{x+3} \geq 0$), and this becomes the domain of f^{-1}. Interchange x and y:

$$x = 1 + \sqrt{y+3}$$

$$\sqrt{y+3} = x - 1$$

$$y = (x-1)^2 - 3 \quad \text{valid because } x - 1 \geq 0$$

$f^{-1}(x) = (x-1)^2 - 3$, domain $x \geq 1$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Finds $f^{-1}(x)$ and gives a valid reason why the inverse exists, or finds $f^{-1}(x)$ with its domain, or equivalent merit	2
Interchanges x and y and makes progress, or explains why the inverse exists, or equivalent merit	1

Without the domain $x \geq 1$, $(x-1)^2 - 3$ is a whole parabola, which is not the inverse. The domain of f^{-1} is the range of f.

13 (c) (ii)

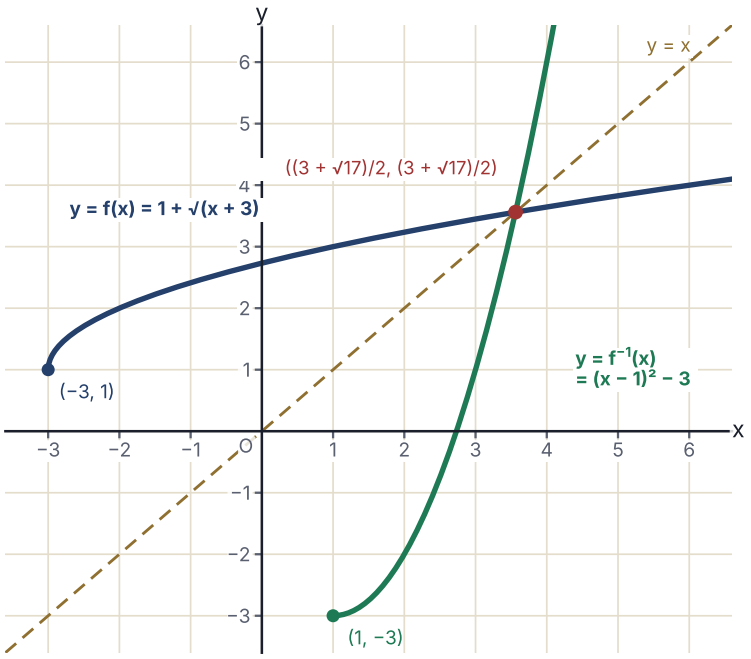
f is increasing, so the graphs of f and f^{-1} can only meet on the line $y = x$. (If $f(a) = b$ and $f(b) = a$ with $a < b$, then $f(a) < f(b)$ gives $b < a$, which is impossible.) Solve $f(x) = x$:

$$1 + \sqrt{x+3} = x \implies \sqrt{x+3} = x - 1 \quad \text{needs } x \geq 1$$

$$x + 3 = x^2 - 2x + 1 \implies x^2 - 3x - 2 = 0 \quad \text{square}$$

$$x = \frac{3 \pm \sqrt{17}}{2}$$

$\dfrac{3 - \sqrt{17}}{2} \approx -0.56$ is less than 1, so it does not satisfy $\sqrt{x+3} = x - 1$ and is rejected.



Question 13 (c) · $y = f(x), y = f^{-1}(x)$ and the line $y = x$

$$\left(\frac{3+\sqrt{17}}{2}, \frac{3+\sqrt{17}}{2}\right)$$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Obtains $x^2 - 3x - 2 = 0$ from $f(x) = x$ or $f^{-1}(x) = x$, or equivalent merit	1

Squaring creates the extra root $\frac{3-\sqrt{17}}{2}$. Check every root against the domain $x \geq 1$of f^{-1}.

13 (d) (i)

$$R\cos(\theta - \alpha) = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$$

$$R\cos\alpha = 12, \quad R\sin\alpha = 5 \quad \text{match coefficients}$$

$$R = \sqrt{12^2 + 5^2} = 13, \quad \tan\alpha = \frac{5}{12}$$

$$\alpha \approx 22^\circ\,37'$$

$$12\cos\theta + 5\sin\theta = 13\cos(\theta - 22^\circ\,37')$$

MARKING GUIDELINES

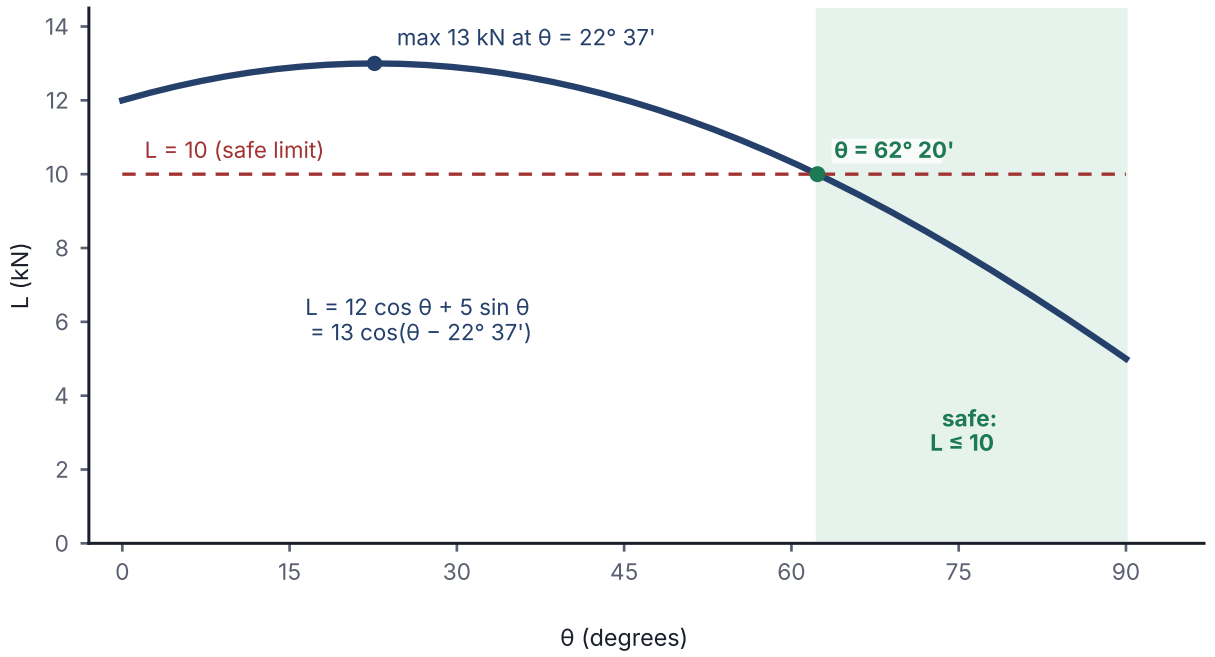
CRITERIA	MARKS
Provides correct solution	2
Finds $R = 13$ or α, or equivalent merit	1

13 (d) (ii)

The strut is safe when $13\cos(\theta-\alpha)\le 10$ that is $\cos(\theta-\alpha)\le\dfrac{10}{13}$.

For $0^\circ\le\theta\le 90^\circ$, $\theta-\alpha$ runs from $-22^\circ 37'$ to $67^\circ 23'$. Cosine equals $\frac{10}{13}$ at $\pm 39^\circ 43'$, and on this interval it is at most $\frac{10}{13}$ only when $\theta-\alpha\ge 39^\circ 43'$ (the negative end only reaches $-22^\circ 37'$, where the cosine is about 0.92).

$$\theta-22^\circ 37'\ge 39^\circ 43' \implies \theta\ge 62^\circ 20'$$



Question 13 (d) · the load on the strut and the safe range

The strut is safe for $62^\circ 20'\le\theta\le 90^\circ$.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Finds the boundary angle $62^\circ 20'$, or equivalent merit	2
Obtains $\cos(\theta-\alpha)\le\frac{10}{13}$ or $\cos(\theta-\alpha)=\frac{10}{13}$, or equivalent merit	1

The equation $\cos(\theta-\alpha)=\frac{10}{13}$ also gives $\theta-\alpha=-39^\circ 43'$, so $\theta=-17^\circ 6'$, which is outside the model's domain. The answer is a range of angles, not a single angle: at $\theta=0^\circ$ the load is 12 kN, which is unsafe.

Question 14

14 (a)

Four people from three professions with at least one of each: one profession supplies two people. Count the three cases.

Engineers, architects, planners	Number of panels
$2,1,1$	${}^4C_2\times{}^3C_1\times{}^2C_1=6\times 3\times 2=36$
$1,2,1$	${}^4C_1\times{}^3C_2\times{}^2C_1=4\times 3\times 2=24$
$1,1,2$	${}^4C_1\times{}^3C_1\times{}^2C_2=4\times 3\times 1=12$

Total: $36 + 24 + 12 = \boxed{72}$ panels.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Correctly counts two of the three cases, or equivalent merit	2
Identifies the three cases, or correctly counts one case, or equivalent merit	1

The tempting shortcut $4 \times 3 \times 2 \times {}^{6}C_{1} = 144$ (one of each, then any fourth person) counts every panel twice: the two people from the same profession can be picked in either order.

14 (b) (i)

After folding, the base is $(16 - 2x)$ cm by $(10 - 2x)$ cm and the height is x cm:

$$V = x(16 - 2x)(10 - 2x) = x\left(160 - 52x + 4x^2\right) = 4x^3 - 52x^2 + 160x$$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	1

14 (b) (ii)

$$4(x-2)^2(x-9) = 4(x^2 - 4x + 4)(x - 9)$$

$$= 4\left(x^3 - 13x^2 + 40x - 36\right)$$

$$= 4x^3 - 52x^2 + 160x - 144 = V - 144$$

(Alternatively: $V(2) - 144 = 0$, so $x - 2$ is a factor of $V - 144$, and division gives $4(x^2 - 11x + 18) = 4(x - 2)(x - 9)$ as the other factor.)

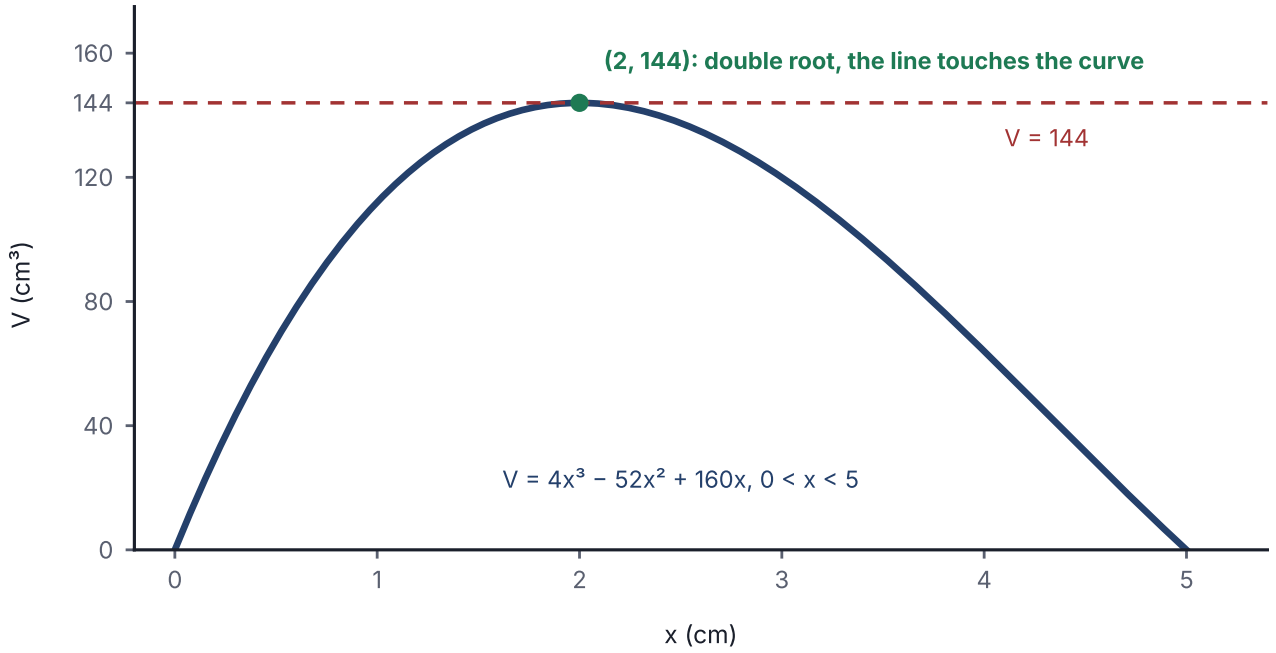
MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Correctly expands $(x-2)^2(x-9)$, or uses the factor theorem to find the factor $x - 2$, or equivalent merit	1

14 (b) (iii)

The sides must have positive length, so $x > 0$ and $10 - 2x > 0$: $0 < x < 5$.

For these x, $(x-2)^2 \geq 0$ and $x - 9 < 0$, so $V - 144 = 4(x-2)^2(x-9) \leq 0$ Hence $V \leq 144$ for every possible tray, with equality only when $x = 2$.



Question 14 (b) · the double root at $x = 2$: the line $V = 144$ touches the curve

When $x = 2$ the tray is 12 cm by 6 cm by 2 cm, and its volume is $12 \times 6 \times 2 = 144$ cm^3.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Shows that $V - 144 \leq 0$for $0 < x < 5$ with a reason, or equivalent merit	2
States $0 < x < 5$, or notes the signs of $(x-2)^2$ and $x - 9$, or finds the dimensions, or equivalent merit	1

The double root is the point: the horizontal line $V = 144$ meets the cubic at $x = 2$ without crossing it, so 144 cm^3 is the greatest volume. No calculus is needed. The root $x = 9$ is outside $0 < x < 5$ and gives no tray.

14 (c) (i)

$(1 - x^2)^n = \sum_{k=0}^{n} {}^nC_k \left(-x^2\right)^k = \sum_{k=0}^{n} (-1)^k \, {}^nC_k \, x^{2k}$, so the coefficient of x^{2k} is $(-1)^k \, {}^nC_k$.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct answer	1

14 (c) (ii)

Write $(1 - x)^n = \sum_{r=0}^{n} (-1)^r \, {}^nC_r \, x^r$ and $(1 + x)^n = \sum_{s=0}^{n} {}^nC_s \, x^s$. A term in x^n in the product comes from x^r in the first and x^{n-r} in the second, for $r = 0, 1, \ldots, n$:

$$\text{coefficient of } x^n \text{ in } (1-x)^n(1+x)^n = \sum_{r=0}^{n} (-1)^r \, {}^nC_r \, {}^nC_{n-r} = \sum_{r=0}^{n} (-1)^r \left({}^nC_r\right)^2 \quad {}^nC_{n-r} = {}^nC_r$$

With $n = 2m$, $x^n = x^{2m}$, and by part (i) with $k = m$ its coefficient in $\left(1 - x^2\right)^{2m}$ is $(-1)^m \, {}^{2m}C_m$. The two expressions are equal for all x, so their coefficients are equal:

$$\sum_{r=0}^{2m} (-1)^r \left({}^{2m}C_r\right)^2 = (-1)^m \, {}^{2m}C_m$$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct proof	3
Obtains the coefficient of x^n in $(1-x)^n(1+x)^n$ as $\sum(-1)^r \left({}^nC_r\right)^2$, or equivalent merit	2
Recognises that the coefficient of x^n in the product is $\sum(-1)^r \, {}^nC_r \, {}^nC_{n-r}$, or equivalent merit	1

Check with $m = 1$: $1 - 4 + 1 = -2 = (-1)^1 \, {}^2C_1$. A two second check like this catches a missing $(-1)^m$.

14 (c) (iii)

By comparing coefficients. The argument of part (ii) shows that the sum is the coefficient of x^n in $\left(1-x^2\right)^n$. That expansion contains only even powers of x, so when n is odd the coefficient of x^n is 0.

By symmetry. Pair the term r with the term $n-r$. Since ${}^nC_{n-r} = {}^nC_r$, the two terms are $(-1)^r \left({}^nC_r\right)^2$ and $(-1)^{n-r} \left({}^nC_r\right)^2$. When n is odd, r and $n-r$ have opposite parity, so the terms cancel, and as $r \neq n-r$ every term has a partner. The sum is 0.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct explanation	2
Notes that $\left(1-x^2\right)^n$ has only even powers of x, or pairs terms using ${}^nC_r = {}^nC_{n-r}$, or equivalent merit	1

Either argument earns full marks. The pairing argument must say why no term is left unpaired: $r = n - r$ would need $n = 2r$, which is even.

What this paper examined

Focus area	Section I	Section II	Marks	Target
Further work with functions	2	13	**15**	15
Polynomials	2	12	**14**	14
Further trigonometry	2	14	**16**	16
Permutations and combinations	2	11	**13**	13
The binomial theorem	2	10	**12**	12
Total	**10**	**60**	**70**	**70**

Where each focus area was examined:

- **Further work with functions (ME1-11-01):** Questions 2 and 4; 11 (b) (iii) cubic inequality; 12 (b) rational inequality; 13 (a) reciprocal graph; 13 (c) inverse function and its intersection with the function.
- **Polynomials (ME1-11-02):** Questions 1 and 10; 11 (b) factor theorem; 12 (e) sums and products of roots; 14 (b) multiplicity and a double root as a tangent.
- **Further trigonometry (ME1-11-03):** Questions 6 and 9; 11 (d) and (e) compound and double angles; 12 (d) three dimensions with bearings; 13 (d) the auxiliary angle in a model.

- **Permutations and combinations (ME1-11-04):** Questions 5 and 8; 11 (c) arrangements with a block; 12 (c) and 13 (b) probability by counting; 14 (a) selection by cases.
- **The binomial theorem (ME1-11-05):** Questions 3 and 7; 11 (a) term independent of x; 12 (a) a coefficient in a product; 14 (c) an identity by comparing coefficients and by symmetry.

The questions rise in difficulty through the paper, and within each question the last parts are the hardest. Use the table to find the focus area where you lost the most marks, and work through that focus area's Practice Questions before you sit Practice Paper 2.

End of worked solutions.

About this resource

MA99 EDUCATION

Scope

NSW Mathematics Extension 1, Year 11. Written to the NSW Mathematics Extension 1 11-12 Syllabus (2024), all Year 11 focus areas, for students sitting the HSC in 2028 and later years. Every value and solution was re-derived independently by computer algebra before printing. Contexts in the questions are invented unless a source is named.

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