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Coverage

Every Year 11 focus area and outcome is in this table, and nothing in the Year 11 course sits outside it. Working mathematically (MAO-WM-01) runs through every chapter. The syllabus is the Mathematics Advanced 11-12 Syllabus (2024), studied in Year 11 from 2026 and first examined in the 2027 HSC; a student in Year 11 in 2027 sits the HSC in 2028. Content the 2017 syllabus placed in Year 11 but the 2024 syllabus moved (discrete probability distributions to Year 12) is not in this book; graph transformations, which moved into Year 11, is Chapter 2.

CHAPTER 1 · FUNCTIONS · MAV-11-01, MAV-11-02

1. Working with Functions

Every later chapter in this course is about a function: a trigonometric one, an exponential one, the derivative of one. This chapter builds the algebra you manipulate them with and the language you describe them in, and it is the longest chapter in the book because it carries the most.

Year 11 Mathematics Advanced · written to the NSW Mathematics Advanced 11-12 Syllabus (2024), focus area Working with functions, first examined in the 2027 HSC.

The focus area has eleven content groups, and the chapter takes them in NESA's order. The first five sections are algebra: indices, factorising, fractions, surds and the quadratic equation. Everything after that is about functions: what one is, the families you must recognise on sight (linear, quadratic, cubic, reciprocal, circle, absolute value), how to build one from a description, and the properties that let you sketch and test them.

NESA content group	Sections	Practice
Algebraic techniques	1.1 to 1.5	Exercise 1A
Introduction to functions and relations	1.6, 1.7	Exercise 1B
Linear functions	1.8, 1.9	Exercise 1B
Quadratic and cubic functions; reciprocal functions	1.10 to 1.14	Exercise 1C
Constructing and using functions; direct and inverse variation; circles and semicircles	1.15 to 1.17	Exercise 1D
Properties of functions, relations and graphs; piecewise-defined functions; absolute value functions	1.18 to 1.20	Exercise 1E

What changed in 2024

New in Year 11. Piecewise-defined functions are now a content group of their own, and continuity is defined here informally, by drawing without lifting the pen (the 2017 course met continuity in calculus). Variation is written in general form, $y = kx^n$ and $y = \dfrac{k}{x^n}$. All four semicircles are named, including $x = \pm\sqrt{r^2 - y^2}$.

Gone from this focus area. General polynomials (degree, coefficients, graphs of any degree) are in Mathematics Extension 1. Sums, differences, products and quotients of functions are no longer in Year 11 Mathematics Advanced. Translations, reflections and dilations, and circles not centred at the origin, are Chapter 2, Graph transformations.

1.1 Index laws

An index counts repeated multiplication, and every index law is that sentence applied once. $a^m \times a^n$ is m copies of a followed by n more, so $m + n$ copies. Dividing cancels n of the m copies. Raising to a power repeats the whole block. The laws for zero, negative and fractional indices are not new facts: they are the only definitions that keep those three laws true.

The index laws ($a, b \neq 0$, m and n rational)

$$a^m \times a^n = a^{m+n} \qquad a^m \div a^n = a^{m-n} \qquad (a^m)^n = a^{mn}$$

$$(ab)^n = a^n b^n \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \qquad a^0 = 1$$

$$a^{-n} = \frac{1}{a^n} \qquad a^{\frac{1}{n}} = \sqrt[n]{a} \qquad a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

Why $a^0 = 1$: $a^n \div a^n$ is 1, and the division law calls it $a^{n-n} = a^0$. Why $a^{-n} = \dfrac{1}{a^n}$: $a^0 \div a^n = a^{-n}$. Why $a^{\frac{1}{2}} = \sqrt{a}$: squaring it gives $a^1 = a$. For an even root, a must be non-negative.

A negative index means reciprocal, never a negative number: $2^{-3} = \dfrac{1}{8}$. A fractional index means a root, and the denominator of the index names the root. When you evaluate one by hand, take the root first and the power second, so the numbers stay small.

Worked example 1

3 MARKS

Simplify $(2a^3b^{-2})^4 \div (4a^5b^{-3})$, writing your answer with positive indices.

01 Raise every factor to the fourth power: $(2a^3b^{-2})^4 = 16a^{12}b^{-8}$. The 2 is a factor too: $2^4 = 16$

02 Divide, subtracting indices: $16a^{12}b^{-8} \div 4a^5b^{-3} = 4a^7b^{-5}$. $-8 - (-3) = -5$

03 Clear the negative index: $\dfrac{4a^7}{b^5}$.

Every factor.
The power outside a bracket reaches the number in front as well as the letters.

Minus a minus.
Subtracting a negative index adds. Write the subtraction out before you simplify it.

WHERE THE MARKS ARE

1 for raising every factor, coefficient included, to the power 4. **1** for the indices a^7 and b^{-5}. **1** for the final form with positive indices. Leaving the 2 as 2 instead of 16 is the most common lost mark.

Worked example 2

3 MARKS

Evaluate without a calculator: (a) $27^{\frac{2}{3}}$ (b) $8^{-\frac{2}{3}}$ (c) $\left(\frac{16}{81}\right)^{-\frac{3}{4}}$.

Flip, root, power. With a negative fractional index on a fraction, do the three moves in that order.

01 (a) $27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = 3^2 = \boxed{9}$. root first, then power

02 (b) $8^{-\frac{2}{3}} = \dfrac{1}{\left(\sqrt[3]{8}\right)^2} = \dfrac{1}{2^2} = \boxed{\dfrac{1}{4}}$.

03 (c) A negative index on a fraction turns it over: $\left(\frac{16}{81}\right)^{-\frac{3}{4}} = \left(\frac{81}{16}\right)^{\frac{3}{4}} = \left(\frac{3}{2}\right)^3 = \boxed{\frac{27}{8}}$. $\sqrt[4]{81} = 3, \sqrt[4]{16} = 2$

WHERE THE MARKS ARE

1 for each value. In (c) the mark needs both moves: the reciprocal for the minus sign and the fourth root for the 4.

Worked example 3

4 MARKS

A cube has volume V cm³. (a) Show that its total surface area is $\boxed{S = 6V^{\frac{2}{3}}}$ cm². (b) Find S when $V = 125$. (c) Find V when $S = 96$.

Undo with the reciprocal. To undo a power two thirds, raise to the power three halves.

Watch the sign. Without the context, the equation with index two thirds equal to 16 has two answers, plus and minus 64.

01 (a) The side length is $s = V^{\frac{1}{3}}$, since $s^3 = V$.

02 Six square faces: $S = 6s^2 = 6\left(V^{\frac{1}{3}}\right)^2 = 6V^{\frac{2}{3}}$, as required.

03 (b) $S = 6 \times 125^{\frac{2}{3}} = 6 \times 5^2 = \boxed{150 \text{ cm}^2}$.

04 (c) $6V^{\frac{2}{3}} = 96$, so $V^{\frac{2}{3}} = 16$. Raise both sides to the power $\frac{3}{2}$: $V = 16^{\frac{3}{2}} = 4^3 = \boxed{64 \text{ cm}^3}$. a volume is positive, so the positive root is the only one

WHERE THE MARKS ARE

1 for the side length as $V^{\frac{1}{3}}$, **1** for the surface area shown, **1** for 150, **1** for 64 with the power $\frac{3}{2}$ applied to both sides.

1.2 Expanding, factorising and algebraic fractions

Expanding and factorising are the same identity read in opposite directions. Expanding is mechanical. Factorising asks you to recognise a pattern, which is why it is what gets examined. Take out the highest common factor first, every time, before you look for anything else.

The patterns to know on sight

$$(a+b)^2 = a^2 + 2ab + b^2 \qquad (a-b)^2 = a^2 - 2ab + b^2 \qquad a^2 - b^2 = (a-b)(a+b)$$

Grouping. Four terms: pair them so each pair leaves the same bracket. **Trinomials** $ax^2 + bx + c$: find two numbers with product ac and sum b, split the middle term, then group.

Three things that are NOT identities, and that cost marks every year: $(a+b)^2 \neq a^2 + b^2$, $\dfrac{1}{a+b} \neq \dfrac{1}{a} + \dfrac{1}{b}$, $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$. Test any of them with $a = b = 1$ to see it fail.

Worked example 4

3 MARKS

Factorise fully: (a) $2x^3 + x^2 - 18x - 9$ (b) $6x^2 - 5x - 6$.

01 (a) Group in pairs: $x^2(2x + 1) - 9(2x + 1)$. take -9 out, not 9

02 Common bracket, then the difference of two squares: $(2x + 1)(x^2 - 9) = (2x + 1)(x - 3)(x + 3)$.

03 (b) Product $6 \times (-6) = -36$ and sum -5: the numbers are -9 and 4.

04 $6x^2 - 9x + 4x - 6 = 3x(2x - 3) + 2(2x - 3) = (2x - 3)(3x + 2)$.

Fully means fully.
After every factorisation, look inside each bracket for another pattern.

Expand to check.
Multiplying your answer back out takes ten seconds and catches every sign slip.

WHERE THE MARKS ARE

1 for a grouping that produces the same bracket twice, **1** for factorising $x^2 - 9$ (stopping at $(2x + 1)(x^2 - 9)$ is not fully factorised), **1** for (b).

Algebraic fractions follow the rules of numerical fractions exactly. Factorise every numerator and denominator first, because you may cancel only a factor common to the whole numerator and the whole denominator, never a term. To add or subtract, use the lowest common denominator; to divide, multiply by the reciprocal.

Worked example 5

4 MARKS

Simplify (a) $\dfrac{a^3b - ab^3}{a^2 + 2ab + b^2}$ (b) $\dfrac{m^3 + m^2}{x^2 - x} \div \dfrac{m + 1}{x - x^3}$.

01 (a) $\dfrac{ab(a^2 - b^2)}{(a + b)^2} = \dfrac{ab(a - b)(a + b)}{(a + b)^2} = \dfrac{ab(a - b)}{a + b}$.

02 (b) Multiply by the reciprocal and factorise: $\dfrac{m^2(m + 1)}{x(x - 1)} \times \dfrac{x(1 - x)(1 + x)}{m + 1}$.

03 Cancel $m + 1$ and x, and use $\dfrac{1 - x}{x - 1} = -1$: the result is $-m^2(x + 1)$.

$1 - x = -(x - 1)$

Factors, not terms.
Cancel a bracket only when it multiplies everything above and everything below.

Reversed brackets.
One minus x is minus one times x minus one. Spot reversed brackets and cancel them to minus one.

WHERE THE MARKS ARE

1 for factorising (a) fully, **1** for its simplified form, **1** for inverting and factorising in (b), **1** for handling $\dfrac{1 - x}{x - 1}$ as -1.

Worked example 6

3 MARKS

Simplify $\dfrac{x^2}{x^2 + 3x + 2} - \dfrac{x}{x + 1}$.

01 Factorise: $x^2 + 3x + 2 = (x + 1)(x + 2)$, so the lowest common denominator is $(x + 1)(x + 2)$.

02 $\dfrac{x^2}{(x + 1)(x + 2)} - \dfrac{x(x + 2)}{(x + 1)(x + 2)} = \dfrac{x^2 - x^2 - 2x}{(x + 1)(x + 2)}$. the minus reaches both terms

Bracket the second numerator.
Write $x(x + 2)$ in brackets before subtracting it. The minus sign must reach every term.

03 $= \dfrac{-2x}{(x+1)(x+2)}$.

WHERE THE MARKS ARE

1 for the common denominator, **1** for the numerator with the subtraction distributed, **1** for the simplified answer.

1.3 Surds and rationalising the denominator

A surd is an irrational root such as $\sqrt{2}$ or $\sqrt[3]{5}$. $\sqrt{9}$ is not a surd, because it is 3. Recall what the symbol means: $\sqrt{x}$ is undefined (as a real number) for $x < 0$, $\sqrt{0} = 0$, and for $x > 0$, $\sqrt{x}$ is the **positive** square root of x. So $\sqrt{9} = 3$, never ± 3. In this course exact form means surd form, and a decimal earns nothing when an exact answer was asked for.

The surd rules ($a, b \geq 0$)

$$\sqrt{a}\,\sqrt{b} = \sqrt{ab} \qquad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}\ (b \neq 0) \qquad \left(\sqrt{a}\right)^2 = a$$

Simplify by removing the largest square factor: $\sqrt{72} = \sqrt{36}\sqrt{2} = 6\sqrt{2}$. Only like surds collect: $5\sqrt{2} + 3\sqrt{2} = 8\sqrt{2}$, but $5\sqrt{2} + 3\sqrt{5}$ does not combine.

Conjugates. $\sqrt{a} + \sqrt{b}$ and $\sqrt{a} - \sqrt{b}$ are conjugates, and their product is rational: $\left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{a} - \sqrt{b}\right) = a - b$. To rationalise a denominator $\sqrt{b} \pm \sqrt{c}$, multiply top and bottom by its conjugate.

Worked example 7

3 MARKS

Simplify (a) $\sqrt{50} + \sqrt{18} - \sqrt{8}$ (b) $\left(2\sqrt{3} - \sqrt{5}\right)^2$.

01 (a) $5\sqrt{2} + 3\sqrt{2} - 2\sqrt{2} = 6\sqrt{2}$. largest square factors 25, 9 and 4

02 (b) $(2\sqrt{3})^2 - 2(2\sqrt{3})(\sqrt{5}) + (\sqrt{5})^2 = 12 - 4\sqrt{15} + 5$.

03 $= 17 - 4\sqrt{15}$.

WHERE THE MARKS ARE

1 for (a). **1** for squaring $2\sqrt{3}$ to 12 (not 6), **1** for the middle term $-4\sqrt{15}$.

Square the coefficient.
Two root three, squared, is four times three. The 2 is squared as well.

Worked example 8

4 MARKS

Express with a rational denominator: (a) $\dfrac{4}{\sqrt{7} + \sqrt{3}}$ (b) $\dfrac{\sqrt{6} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$.

01 (a) Multiply by $\dfrac{\sqrt{7} - \sqrt{3}}{\sqrt{7} - \sqrt{3}}$: the denominator becomes $7 - 3 = 4$.

02 $\dfrac{4\left(\sqrt{7} - \sqrt{3}\right)}{4} = \sqrt{7} - \sqrt{3}$.

03 (b) Multiply by $\dfrac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}}$: the denominator is $3 - 2 = 1$.

Flip the sign, not the order.
The conjugate of root 3 minus root 2 is root 3 plus root 2.

Denominator first.
Work out the new denominator before touching the numerator. Often it is 1 and the rest is easy.

04 Numerator: $\sqrt{18}+\sqrt{12}+\sqrt{6}+\sqrt{4}=3\sqrt{2}+2\sqrt{3}+\sqrt{6}+2$, so the answer is $2+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$.
four products, then simplify each surd

WHERE THE MARKS ARE

1 for the conjugate in (a), **1** for its simplified answer. **1** for the conjugate and denominator in (b), **1** for all four numerator terms simplified.

1.4 Solving quadratic equations

A quadratic equation $ax^2+bx+c=0$ with $a\neq 0$ can be solved three ways, and the question often chooses for you. **Factorising** is fastest when it works, and with rational coefficients it works exactly when the roots are rational. **Completing the square** rewrites the quadratic so x appears only once. The **quadratic formula** always works, and it is completing the square done once in general.

The quadratic formula, by completing the square

Divide $ax^2+bx+c=0$ by a, move $\dfrac{c}{a}$ across, and add $\left(\dfrac{b}{2a}\right)^2$ to both sides:

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}=\frac{b^2-4ac}{4a^2}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a},\qquad a\neq 0$$

Completing the square on x^2+px: add the square of half the coefficient of x. $x^2+px+\left(\dfrac{p}{2}\right)^2=\left(x+\dfrac{p}{2}\right)^2$.

Worked example 9

5 MARKS

Solve (a) $6x^2-5x-6=0$ by factorising (b) $x^2+6x+4=0$ by completing the square (c) $2x^2-3x-4=0$ using the quadratic formula.

01 (a) $(2x-3)(3x+2)=0$ from Worked example 4, so $x=\dfrac{3}{2}$ or $x=-\dfrac{2}{3}$.

02 (b) $x^2+6x=-4$. Add $3^2=9$: $(x+3)^2=5$. half of 6, squared

03 $x+3=\pm\sqrt{5}$, so $x=-3\pm\sqrt{5}$.

04 (c) $a=2, b=-3, c=-4$: $x=\dfrac{3\pm\sqrt{9+32}}{4}=\dfrac{3\pm\sqrt{41}}{4}$. $-b=3$ and $-4ac=+32$

Both signs.
Taking a square root of both sides gives plus or minus. Dropping the minus loses a root and a mark.

Brackets for negatives.
Substitute b equals minus 3 as (minus 3). Then minus b is plus 3 and b squared is plus 9.

WHERE THE MARKS ARE

1 for (a). **1** for adding 9 to both sides in (b), **1** for both roots with ±. **1** for correct substitution in (c), **1** for the simplified roots.

1.5 The discriminant

The part of the formula under the root decides what kind of roots the equation has before you find them. It is called the discriminant, $\Delta=b^2-4ac$, and the roots are $x=\dfrac{-b\pm\sqrt{\Delta}}{2a}$.

Discriminant	Roots of $ax^2 + bx + c = 0$
$\Delta > 0$	two distinct (different) real roots
$\Delta = 0$	equal roots: one repeated real root, $x = -\dfrac{b}{2a}$
$\Delta < 0$	no real roots
$\Delta \geq 0$	real roots (distinct or equal)
Δ a perfect square, a, b, c rational	rational roots

Language matters here. An **equation** has roots; a **function** has zeroes. The roots of $x^2 - 4 = 0$ are ± 2, and ± 2 are the zeroes of the function $f(x) = x^2 - 4$.

Worked example 10

3 MARKS

Find k if the equation $(2k+3)x^2 - 4kx + 4 = 0$ has equal roots.

01 Equal roots means $\Delta = 0$ $(-4k)^2 - 4(2k+3)(4) = 0$.

02 $16k^2 - 32k - 48 = 0$, so $16(k^2 - 2k - 3) = 16(k-3)(k+1) = 0$.

03 $k = 3$ or $k = -1$. Both give $2k + 3 \neq 0$, so the equation is quadratic in each case. the leading coefficient must not vanish

Name a, b and c.
Write them down before substituting. With k in the coefficients, this is where errors start.

Check a is not zero.
If a value of k made the x squared term vanish, the equation would no longer be quadratic.

WHERE THE MARKS ARE

1 for setting $\Delta = 0$ with the correct a, b and c, **1** for the quadratic in k, **1** for both values. Writing $b = 4k$ instead of $-4k$ happens to give the same b^2, but write the sign anyway.

Worked example 11

3 MARKS

Show that the roots of $kx^2 + (2k+1)x + k + 1 = 0$ are real and rational for every rational $k \neq 0$, and find them.

01 $\Delta = (2k+1)^2 - 4k(k+1)$, which is $4k^2 + 4k + 1 - 4k^2 - 4k = 1$.

02 $\Delta = 1 > 0$ and $1 = 1^2$ is a perfect square, so the roots are real, distinct and rational.

03 $x = \dfrac{-(2k+1) \pm 1}{2k}$, which gives $x = -1$ or $x = -\dfrac{k+1}{k}$.

Rational needs a square.
A positive discriminant gives real roots. Only a perfect square makes them rational.

WHERE THE MARKS ARE

1 for $\Delta = 1$, **1** for the conclusion naming both reasons (positive and a perfect square), **1** for the roots.

The trap · strict or not

A discriminant condition is about an **equation** $ax^2 + bx + c = 0$ with $a \neq 0$. When the leading coefficient contains the parameter, check the values you find do not make it zero. And $\Delta \geq 0$ means real roots, while $\Delta > 0$ means two **distinct** real roots: read which one the question asks for.

Exercise 1A · Algebraic techniques

FOUNDATION One idea, applied once. The point of them is to check the idea is there.

1 Simplify (a) $a^5 \times a^{-2}$ (b) $(3x^2)^3$ (c) $12x^5 \div 4x^{-1}$. (3 marks)

2 Evaluate (a) $16^{\frac{3}{4}}$ (b) $25^{-\frac{1}{2}}$ (c) $\left(\dfrac{8}{27}\right)^{-\frac{2}{3}}$. (3 marks)

3 Simplify (a) $\sqrt{75}$ (b) $\sqrt{12} + \sqrt{27}$ (c) $3\sqrt{2} \times 4\sqrt{6}$. (3 marks)

4 Solve (a) $x^2 - 5x - 14 = 0$ (b) $3x^2 + 4x - 4 = 0$. (2 marks)

STANDARD Two steps, or one step plus a decision. Most of the examination lives here.

5 Factorise fully (a) $3x^3 - 12x$ (b) $x^3 - 2x^2 - 9x + 18$ (c) $12x^2 + x - 6$. (3 marks)

6 Rationalise the denominator (a) $\dfrac{6}{\sqrt{3}}$ (b) $\dfrac{5}{\sqrt{7} - \sqrt{2}}$ (c) $\dfrac{\sqrt{3} + 1}{\sqrt{3} - 1}$. (3 marks)

7 (a) Solve $x^2 - 4x - 7 = 0$ by completing the square. (b) Solve $x^{\frac{3}{2}} = 27$. (3 marks)

8 Use the discriminant to state the number and type of real roots of (a) $3x^2 - 2x + 5 = 0$ (b) $4x^2 - 12x + 9 = 0$ (c) $2x^2 + 7x - 3 = 0$. (3 marks)

ADVANCED Three or more steps, or a method you must choose before you can start.

9 Simplify $\dfrac{x^2 - 9}{x^2 + x - 6} \times \dfrac{x^2 - 4}{x^2 - 5x + 6}$. (3 marks)

10 Simplify $\dfrac{3}{x - 2} - \dfrac{2}{x + 1} - \dfrac{9}{(x - 2)(x + 1)}$. (3 marks)

11 Find the values of k for which $x^2 + kx + k + 3 = 0$ has equal roots. (3 marks)

12 Show that the roots of $x^2 + (p - 1)x - p = 0$ are real and rational for every rational p, and find them. (3 marks)

EXAM-STYLE Multi-part, written the way the HSC writes them. Attempt these last.

13 (a) Show that $\dfrac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} = 4 + \sqrt{15}$. (b) Hence evaluate $\dfrac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} + \dfrac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}$. (4 marks)

14 Consider $2x^2 - (k + 2)x + k = 0$. (a) Show that $\Delta = (k - 2)^2$. (b) Hence explain why the roots are real for every real k, and find the value of k for which they are equal. (c) Solve the equation when $k = 6$. (5 marks)

1.6 Relations and functions

A **relation** between two sets links elements of the first set with elements of the second. It can be written as a set of ordered pairs (x, y), with x taken from the first set and y an element of the second set that x is linked to. Take the set of students in a class and the set of months, and relate each student to the month of their birth. Every student is paired with exactly one month. Read the other way, from month to student, a month may be paired with several students or with none.

Definition · Function

A **function** f is a relation that pairs every element x of the first set with **exactly one** element y in the second set. In this course both sets hold real numbers: f is a real function of a real variable.

Every function is a relation. A relation is a function only when no x is paired with two different y values. The student-to-birth-month relation is a function. The month-to-student relation is not.

WORKING SPACE

Chapter 1 worked solutions

EXERCISE 1A

1

(a) $a^{5+(-2)} = a^3$. (b) $3^3(x^2)^3 = 27x^6$. (c) $\frac{12}{4}x^{5-(-1)} = 3x^6$.

$a^3,\ 27x^6,\ 3x^6$

In (c), subtracting a negative index adds. Writing $3x^4$ is the usual slip.

2

(a) $\left(\sqrt[4]{16}\right)^3 = 2^3 = 8$. (b) $\dfrac{1}{\sqrt{25}} = \dfrac{1}{5}$.

(c) $\left(\dfrac{27}{8}\right)^{\frac{2}{3}} = \left(\dfrac{3}{2}\right)^2 = \dfrac{9}{4}$.

$8,\ \dfrac{1}{5},\ \dfrac{9}{4}$

Turn the fraction over for the minus sign before taking the cube root.

3

(a) $\sqrt{25 \times 3} = 5\sqrt{3}$. (b) $2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$.

(c) $12\sqrt{12} = 12 \times 2\sqrt{3} = 24\sqrt{3}$.

$5\sqrt{3},\ 5\sqrt{3},\ 24\sqrt{3}$

Multiply the coefficients and the surds separately, then simplify the surd. $12\sqrt{12}$ is not finished.

4

(a) $(x-7)(x+2) = 0$. (b) $(3x-2)(x+2) = 0$.

(a) $x = 7$ or $x = -2$ (b) $x = \dfrac{2}{3}$ or $x = -2$

Expand your factors to check them before solving.

5

(a) $3x(x^2-4) = 3x(x-2)(x+2)$.

(b) $x^2(x-2) - 9(x-2) = (x-2)(x^2-9) = (x-2)(x-3)(x+3)$.

(c) Product -72, sum 1: 9 and -8. $12x^2 + 9x - 8x - 6 = 3x(4x+3) - 2(4x+3)$.

(a) $3x(x-2)(x+2)$ (b) $(x-2)(x-3)(x+3)$ (c) $(4x+3)(3x-2)$

Common factor first in (a), then the difference of two squares. 'Fully' means no bracket can be factorised further.

6

(a) $\dfrac{6\sqrt{3}}{3} = 2\sqrt{3}$.

(b) $\dfrac{5\left(\sqrt{7}+\sqrt{2}\right)}{7-2} = \sqrt{7}+\sqrt{2}$.

(c) $\dfrac{\left(\sqrt{3}+1\right)^2}{3-1} = \dfrac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$.

$2\sqrt{3},\ \sqrt{7}+\sqrt{2},\ 2+\sqrt{3}$

In (c), divide both terms of $4 + 2\sqrt{3}$ by 2.

7

(a) $x^2 - 4x = 7$. Add 4: $(x-2)^2 = 11$, so $x - 2 = \pm\sqrt{11}$.

(b) $x = 27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = 9$. Check: $9^{\frac{3}{2}} = 3^3 = 27$.

(a) $x = 2 \pm \sqrt{11}$ (b) $x = 9$

In (b), raise both sides to the reciprocal power $\frac{2}{3}$. Only $x \geq 0$is possible, since $x^{\frac{3}{2}}$ needs $\sqrt{x}$.

8

(a) $\Delta = 4 - 60 = -56 < 0$: no real roots.

(b) $\Delta = 144 - 144 = 0$: one real root, repeated (equal roots).

(c) $\Delta = 49 + 24 = 73 > 0$ and 73 is not a perfect square: two distinct irrational real roots.

none; equal roots; two distinct irrational roots

In (c), $-4ac = -4(2)(-3) = +24$. A negative c makes Δ larger, not smaller.

9

$\dfrac{(x-3)(x+3)}{(x+3)(x-2)} \times \dfrac{(x-2)(x+2)}{(x-2)(x-3)}$.

Cancel $(x-3)$, $(x+3)$ and one $(x-2)$.

$\dfrac{x+2}{x-2}$

Factorise all four expressions before cancelling anything.

10

Common denominator $(x-2)(x+1)$: numerator $3(x+1) - 2(x-2) - 9 = 3x + 3 - 2x + 4 - 9 = x - 2$.

$\dfrac{x-2}{(x-2)(x+1)}$, and cancel $x - 2$.

$\dfrac{1}{x+1}$

$-2(x-2) = -2x + 4$. The minus sign reaches the -2 inside the bracket.

11

$\Delta = k^2 - 4(1)(k+3) = k^2 - 4k - 12 = (k-6)(k+2) = 0$.

$k = 6$ or $k = -2$

Equal roots means $\Delta = 0$ exactly. The leading coefficient is 1, so there is nothing to exclude.

12

$\Delta = (p-1)^2 + 4p = p^2 + 2p + 1 = (p+1)^2 \geq 0$, the square of a rational number.

So the roots are real and rational: $x = \dfrac{(1-p) \pm (p+1)}{2}$.

$x = 1$ or $x = -p$

Check by factorising: $x^2 + (p-1)x - p = (x-1)(x+p)$.

13

(a) Multiply top and bottom by $\sqrt{5} + \sqrt{3}$: $\dfrac{\left(\sqrt{5}+\sqrt{3}\right)^2}{5-3} = \dfrac{8 + 2\sqrt{15}}{2} = 4 + \sqrt{15}$.

(b) The second fraction is the reciprocal of the first, and the same method gives $4 - \sqrt{15}$. The sum is $(4 + \sqrt{15}) + (4 - \sqrt{15})$.

(b) 8

'Hence' points to part (a): the second term needs only one sign changed.

14

(a) $\Delta = (k+2)^2 - 4(2)(k) = k^2 + 4k + 4 - 8k = k^2 - 4k + 4 = (k-2)^2$.

(b) A square is never negative, so $\Delta \geq 0$ and the roots are real for every real k (and $a = 2 \neq 0$). They are equal when $(k-2)^2 = 0$, so $k = 2$.

(c) $k = 6$: $2x^2 - 8x + 6 = 0$, $x^2 - 4x + 3 = 0$, $(x-1)(x-3) = 0$.

(b) $k = 2$ (c) $x = 1$ or $x = 3$

A discriminant that is a perfect square is the signal that the roots are always real, and here always rational.

EXERCISE 1B

1

(a) Each input appears once: a function. (b) The input 2 has outputs 3 and -3: not a function.

(c) $x = 3$ is a vertical line: the input 3 has every y as an output, so not a function. (d) $y = 3$ gives the single output 3 for every x: a function.

(a) and (d) are functions; (b) and (c) are not

A vertical line is the one straight line that is not a function.

2

(a) $f(0) = 3$. (b) $f(-1) = 2(1) + 1 + 3 = 6$. (c) $f\left(\frac{1}{2}\right) = 2\left(\frac{1}{4}\right) - \frac{1}{2} + 3 = 3$.

$3, 6, 3$

In (b), $-(-1) = +1$. Brackets round the substituted value make that automatic.

3

$2y = 4x + 7$, so $y = 2x + \frac{7}{2}$.

gradient 2, y-intercept $\frac{7}{2}$

Rearrange to $y = mx + c$ first. Reading $m = 4$ from the general form is wrong.

4

$y - 1 = -2(x - 3)$, so $y = -2x + 7$.

$2x + y - 7 = 0$

General form has everything on one side and equals zero.

5

(a) $x + 3 \geq 0$ domain $[-3, \infty)$ range $[0, \infty)$.

(b) Domain all real x; $x^2 \geq 0$ so range $[1, \infty)$.

(c) Domain $x \neq 1$; range $y \neq 0$.

(a) $[-3, \infty)$ $[0, \infty)$ (b) $(-\infty, \infty)$ $[1, \infty)$ (c) $x \neq 1, y \neq 0$

For (c) the numerator 1 is never zero, which is why 0 is missing from the range.

6

(a) $g(a-1) = (a-1)^2 - 4(a-1) = a^2 - 2a + 1 - 4a + 4$.

(b) $t(t-4) = 0$. (c) $t^2 - 4t - 5 = 0$, $(t-5)(t+1) = 0$.

(a) $a^2 - 6a + 5$ (b) 0 and 4 (c) $t = 5$ or $t = -1$