MA99 EDUCATION

Focus area 2 Introduction to vectors: Summary

02

NSW MATHEMATICS EXTENSION 1 11-12 SYLLABUS 2024 FOR HSC 2027 AND LATER

Notation and 3D

Item	Result
Vector	magnitude and direction; equal vectors have the same magnitude and direction, wherever drawn
Notation	$\mathbf{a}$ printed, $\underset{\sim}{a}$ handwritten, $\overrightarrow{AB}$; magnitude $\lvert\mathbf{a}\rvert$; unit vector $\hat{\mathbf{a}}$; zero vector $\mathbf{0}$ $(\underset{\sim}{0})$
Position vector	$\overrightarrow{OP}$, tail at the origin
Components	$x\mathbf{i}+y\mathbf{j}+z\mathbf{k}=(x,y,z)=\begin{pmatrix}x\\y\\z\end{pmatrix}$
Head minus tail	$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\begin{pmatrix}u-x\\v-y\\w-z\end{pmatrix}$
3D axes	right-handed; coordinate planes xy: $z=0$, xz: $y=0$, yz: $x=0$
Midpoint, distance	$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$, $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Operations

Operation	Rule	Geometry
$k\mathbf{a}$	multiply every component	length $\lvert k\rvert\lvert\mathbf{a}\rvert$; reversed if $k<0$
$\mathbf{a}\pm\mathbf{b}$	add or subtract components	triangle law; parallelogram diagonals
$\lvert\mathbf{a}\rvert$	$\sqrt{x^2+y^2}$, $\sqrt{x^2+y^2+z^2}$	length of the arrow
$\hat{\mathbf{a}}$	$\frac{1}{\lvert\mathbf{a}\rvert}\mathbf{a}$	length 1, same direction
parallel	$\mathbf{a}=k\mathbf{b}$, $k\neq 0$	same or opposite direction
perpendicular (2D)	$\begin{pmatrix}b\\-a\end{pmatrix}$, $\begin{pmatrix}-b\\a\end{pmatrix}$ to $\begin{pmatrix}a\\b\end{pmatrix}$	turned 90°, same length

Direction of a 2D vector: angle from the positive x axis (sketch for the quadrant), or a bearing, clockwise from north, or an angle suited to the context.

The scalar product and projection

$$\mathbf{a}\cdot\mathbf{b}=x_1x_2+y_1y_2+z_1z_2=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta,\quad 0^\circ\leq\theta\leq 180^\circ$$

Use	Formula or test
magnitude	$\mathbf{a}\cdot\mathbf{a}=\lvert\mathbf{a}\rvert^2$, so $\lvert\mathbf{a}\rvert=\sqrt{\mathbf{a}\cdot\mathbf{a}}$

Use	Formula or test
angle	$\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert} = \hat{\mathbf{a}}\cdot\hat{\mathbf{b}}$; sign of $\mathbf{a}\cdot\mathbf{b}$: acute, right, obtuse
perpendicular	$\mathbf{a}\cdot\mathbf{b} = 0$ (non zero vectors)
parallel	$\mathbf{a}\cdot\mathbf{b} = \pm\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert$
projection	$\operatorname{proj}_{\mathbf{b}}\mathbf{a} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{b}\rvert^2}\right)\mathbf{b} = (\mathbf{a}\cdot\hat{\mathbf{b}})\hat{\mathbf{b}}$
perpendicular component	$\mathbf{a} - \operatorname{proj}_{\mathbf{b}}\mathbf{a}$; its dot product with $\mathbf{b}$ is 0

Why the two forms agree: the cosine rule in the triangle with sides $\mathbf{a}$, $\mathbf{b}$, $\mathbf{a}-\mathbf{b}$, with $\lvert\mathbf{a}-\mathbf{b}\rvert^2$ expanded in components. On the reference sheet: the 2D magnitude, the 2D scalar product in both forms, and the projection. Not on it: the 3D versions, $\hat{\mathbf{a}}$, and the angle rearrangement.

Motion in two dimensions

Quantity	Result
position	$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$: parametric equations; eliminate t for the path
displacement	$\mathbf{r}(t_2) - \mathbf{r}(t_1)$; magnitude = distance between the points
constant velocity	$\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$; collide: both components equal at one t
relative velocity	actual = velocity relative to the medium + current (or wind); triangle of velocities
velocity, acceleration	$\mathbf{v} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$, $\mathbf{a} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$, speed $\lvert\mathbf{v}\rvert$
from acceleration	integrate each component; constant vectors from the initial conditions

Projectiles

Point mass, no air resistance, near the Earth's surface: $\mathbf{a} = -g\mathbf{j}$. From height h at speed V and angle θ:

Horizontal	Vertical
$\ddot{x} = 0$	$\ddot{y} = -g$
$\dot{x} = V\cos\theta$	$\dot{y} = V\sin\theta - gt$
$x = Vt\cos\theta$	$y = Vt\sin\theta - \frac{1}{2}gt^2 + h$

$$y = x\tan\theta - \frac{gx^2(1+\tan^2\theta)}{2V^2} + h, \qquad R = \frac{V^2\sin 2\theta}{g} \text{ (level ground)}$$

1. Derive the equations by integrating twice; find each constant at $t = 0$.
2. Time of flight: $y = 0$, positive root. Greatest height: $\dot{y} = 0$.
3. Range: x at the time of flight. Impact: $\mathbf{v}$ at that time; speed and angle below the horizontal.
4. Unknown angle: the trajectory is a quadratic in $\tan\theta$; expect two angles.

The traps, and can I do it

1. Tail minus head for $\overrightarrow{AB}$.
2. Two equal ratios taken as proof of parallel; all components must agree.

3. $|\mathbf{a}+\mathbf{b}|$ written as $|\mathbf{a}|+|\mathbf{b}|$; a negative magnitude.
4. Dividing by $|\mathbf{b}|$ instead of $|\mathbf{b}|^2$ in the projection.
5. The angle in the wrong quadrant from $\tan^{-1}$; no sketch.
6. Crossing paths taken as a collision.
7. Wind from the south read as blowing south.
8. Doubling the time to the top when the launch height is not zero.

- [ ] Use the notation, equal vectors and position vectors (1)
- [ ] Plot in 3D, name coordinate planes, use 3D midpoint and distance (2)
- [ ] Scale, add, subtract; find magnitudes, unit vectors, directions and bearings (3)
- [ ] Test for parallel and perpendicular vectors (3, 4)
- [ ] Find angles, projections and perpendicular components with the scalar product (4)
- [ ] Find paths, displacements, collisions and relative velocities (5)
- [ ] Find velocity and acceleration by calculus, and position by integration (5)
- [ ] Derive the projectile equations and find every feature of a flight (6)

Written to the NSW Mathematics Extension 1 11-12 Syllabus (2024), the syllabus Year 12 studies from Term 4 2026, first examined in the 2027 HSC. Year 11 work is assumed, and HSC questions can draw on it.

About this resource

MA99 EDUCATION

Scope

NSW Mathematics Extension 1, Year 12. Written to the NSW Mathematics Extension 1 11-12 Syllabus (2024), focus area 2, Introduction to vectors (ME1-12-02), for students sitting the HSC in 2027 and later years. Every value, identity and solution was re-derived independently by computer algebra before printing. Contexts in the questions are invented unless a source is named.

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MA99 EDUCATION

Focus area 2 Introduction to vectors: Practice Questions

02

NSW MATHEMATICS EXTENSION 1 11-12 SYLLABUS 2024 FOR HSC 2027 AND LATER

How to use this set

Band	Questions	Marks
Foundation	1 to 7	11
Standard	8 to 11	12
Advanced	12 to 15	12
Exam-style	16 and 17	10

Total 45 marks. Questions 1 to 4 are multiple choice. Use $g = 10$ m/s^2 where needed. Draw a diagram for every question about directions, velocities or projectiles before you calculate.

Questions

FOUNDATION One idea, applied once.

Question 1

1 MARK

The points $P(1, -2, 3)$ and $Q(4, 2, -1)$ are given. Which vector is $\overrightarrow{PQ}$?

A $-3\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$
B $3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}$
C $5\mathbf{i} + 2\mathbf{k}$
D $3\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$

Question 2

1 MARK

Which point lies in the xz-plane?

A $(0, 2, -3)$
B $(2, -3, 0)$
C $(2, 1, -3)$
D $(2, 0, -3)$

Question 3

1 MARK

Non zero vectors $\mathbf{a}$ and $\mathbf{b}$ have $|\mathbf{a}| = 2$, $|\mathbf{b}| = 3$ and $\mathbf{a} \cdot \mathbf{b} = -3$. What is the angle between them?

A 120°
B 60°
C 180°
D 126.9°

Question 4

1 MARK

Which vector is perpendicular to $3\mathbf{i} - 2\mathbf{j}$ and has the same magnitude?

A $-3\mathbf{i} + 2\mathbf{j}$
B $2\mathbf{i} - 3\mathbf{j}$
C $2\mathbf{i} + 3\mathbf{j}$
D $4\mathbf{i} + 6\mathbf{j}$

5 Let $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 3\mathbf{j} - \mathbf{k}$. Find $\mathbf{a} + 2\mathbf{b}$ and $|\mathbf{a}|$. (2 marks)

6 Find the unit vector in the direction of $6\mathbf{i} - 8\mathbf{j}$, and the vector of magnitude 15 in that direction. (2 marks)

7 A particle has position $\mathbf{r} = (1 + 2t)\mathbf{i} + (3 - t)\mathbf{j}$. Find the Cartesian equation of its path, its displacement from $t = 0$ to $t = 3$, and the distance between those two positions. (3 marks)

STANDARD Two steps, or one step plus a decision.

8 Find the angle between $2\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$. (3 marks)

9 For $\mathbf{a} = 3\mathbf{i} + \mathbf{j}$ and $\mathbf{b} = \mathbf{i} - 2\mathbf{j}$, find $\text{proj}_{\mathbf{b}}\,\mathbf{a}$ and the component of $\mathbf{a}$ perpendicular to $\mathbf{b}$, and check your answer. (3 marks)

10 A particle moves with $\mathbf{r} = 3\sin 2t\,\mathbf{i} + 3\cos 2t\,\mathbf{j}$. Find $\mathbf{v}$ and $\mathbf{a}$, show that the speed is constant, and show that $\mathbf{a} = -4\mathbf{r}$. (3 marks)

11 A ball is projected from the ground at 25 m/s at an angle θ with $\tan\theta = \frac{4}{3}$. Starting from $\mathbf{a} = -10\mathbf{j}$, derive $\mathbf{r}(t)$, then find the time of flight and the range. (3 marks)

ADVANCED Three or more steps, or a method you must choose first.

12 Let $\mathbf{u} = m\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = 2\mathbf{i} + (m-1)\mathbf{j} + 4\mathbf{k}$. **(a)** Find m so that $\mathbf{u}$ and $\mathbf{v}$ are perpendicular. **(b)** Show that $\mathbf{u}$ and $\mathbf{v}$ are never parallel. (3 marks)

13 A swimmer can swim at 1.5 m/s in still water. A river 40 m wide flows at 0.9 m/s. In what direction must the swimmer head to land directly opposite, and how long does the crossing take? (3 marks)

14 A particle has acceleration $\mathbf{a} = 2\mathbf{i} - 6t\,\mathbf{j}$, starts at $\mathbf{i}$ with velocity $3\mathbf{j}$. Find $\mathbf{r}(t)$, and its speed when it next crosses the x axis. (3 marks)

15 $A(1, 0, 2)$, $B(3, -1, 4)$ and $C(2, 4, 3)$ are three points. Show that $\angle BAC$ is a right angle and find the exact area of triangle ABC. (3 marks)

EXAM-STYLE Multi-part, written the way the HSC writes them.

Question 16

5 MARKS

A stone is thrown from the top of a building 15 m high at V m/s, at 45° above the horizontal. It lands on level ground 45 m from the base of the building. Take the origin at the base, $g = 10$ m/s^2.

(a) Starting from $\mathbf{a} = -10\mathbf{j}$, show that $\mathbf{r}(t) = \dfrac{Vt}{\sqrt{2}}\mathbf{i} + \left(15 + \dfrac{Vt}{\sqrt{2}} - 5t^2\right)\mathbf{j}$. 2 MARKS

(b) Find the exact value of V. 3 MARKS

Question 17

5 MARKS

Let $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + 4\mathbf{k}$.

(a) Find $\mathbf{a} \cdot \mathbf{b}$, $|\mathbf{a}|$ and $|\mathbf{b}|$. 1 MARK

(b) Find the angle between $\mathbf{a}$ and $\mathbf{b}$, to the nearest 0.1°. 1 MARK

(c) Find $\text{proj}_{\mathbf{b}}\,\mathbf{a}$. 1 MARK

(d) Find the component of $\mathbf{a}$ that is at right angles to $\mathbf{b}$, and verify that it is perpendicular to $\mathbf{b}$. 2 MARKS

WORKING SPACE

Worked solutions · Foundation

1

(B) Head minus tail: $(4-1)\mathbf{i}+(2+2)\mathbf{j}+(-1-3)\mathbf{k}=3\mathbf{i}+4\mathbf{j}-4\mathbf{k}$.

(A) is $\overrightarrow{QP}$, tail minus head. (C) adds the position vectors. (D) loses the sign of the $\mathbf{k}$ component.

2

(D) The xz-plane is $y=0$, and only $(2,0,-3)$ has $y=0$.

(A) is in the yz-plane ($x=0$), (B) in the xy-plane ($z=0$), and (C) in none of them.

3

(A) $\cos\theta=\frac{-3}{2\times3}=-\frac{1}{2}$, so $\theta=120^\circ$.

(B) drops the negative sign. (C) assumes a negative dot product means opposite directions; that needs $\mathbf{a}\cdot\mathbf{b}=-6$. (D) divides by $|\mathbf{a}|+|\mathbf{b}|$: $\cos^{-1}(-0.6)$.

4

(C) $(3)(2)+(-2)(3)=0$, and $|2\mathbf{i}+3\mathbf{j}|=\sqrt{13}=|3\mathbf{i}-2\mathbf{j}|$.

(A) is -1 times the vector: parallel. (B) swaps without changing a sign; its dot product is 12. (D) is perpendicular but twice as long.

5

$\mathbf{a}+2\mathbf{b}=(2+2)\mathbf{i}+(-1+6)\mathbf{j}+(2-2)\mathbf{k}=\boxed{4\mathbf{i}+5\mathbf{j}}$. $|\mathbf{a}|=\sqrt{4+1+4}=\boxed{3}$.

WHERE THE MARKS ARE	
1 mark	$4\mathbf{i}+5\mathbf{j}$
1 mark	$\lvert\mathbf{a}\rvert=3$

6

$|6\mathbf{i}-8\mathbf{j}|=10$, so the unit vector is $\boxed{\frac{3}{5}\mathbf{i}-\frac{4}{5}\mathbf{j}}$, and 15 times it is $\boxed{9\mathbf{i}-12\mathbf{j}}$.

WHERE THE MARKS ARE	
1 mark	Unit vector
1 mark	$9\mathbf{i}-12\mathbf{j}$

7

$t=\frac{x-1}{2}$, $y=3-\frac{x-1}{2}$, so $\boxed{x+2y=7}$. $\mathbf{r}(3)-\mathbf{r}(0)=(7\mathbf{i})-(\mathbf{i}+3\mathbf{j})=\boxed{6\mathbf{i}-3\mathbf{j}}$, distance $\boxed{\sqrt{45}=3\sqrt{5}}$.

WHERE THE MARKS ARE	
1 mark	Cartesian equation
1 mark	Displacement
1 mark	Distance

Worked solutions · Standard

8

Dot product $2-1+2=3$; magnitudes $\sqrt{6}$ and $\sqrt{6}$; $\cos\theta=\frac{3}{6}=\frac{1}{2}$, so $\boxed{\theta=60^\circ}$.

WHERE THE MARKS ARE	
1 mark	Scalar product
1 mark	Magnitudes
1 mark	60°

9

$\mathbf{a}\cdot\mathbf{b}=3-2=1$, $|\mathbf{b}|^2=5$: $\text{proj}_{\mathbf{b}}\,\mathbf{a}=\boxed{\frac{1}{5}\mathbf{i}-\frac{2}{5}\mathbf{j}}$. Perpendicular component $\mathbf{a}-\text{proj}_{\mathbf{b}}\,\mathbf{a}=\boxed{\frac{14}{5}\mathbf{i}+\frac{7}{5}\mathbf{j}}$.

Check: $\left(\frac{14}{5}\mathbf{i}+\frac{7}{5}\mathbf{j}\right)\cdot(\mathbf{i}-2\mathbf{j})=\frac{14}{5}-\frac{14}{5}=0$.

WHERE THE MARKS ARE	
1 mark	Projection
1 mark	Perpendicular component
1 mark	The check

10

$\mathbf{v} = 6\cos 2t\,\mathbf{i} - 6\sin 2t\,\mathbf{j}$, so speed $= \sqrt{36\cos^2 2t + 36\sin^2 2t} = 6$, constant.

$\mathbf{a} = -12\sin 2t\,\mathbf{i} - 12\cos 2t\,\mathbf{j} = -4(3\sin 2t\,\mathbf{i} + 3\cos 2t\,\mathbf{j}) = -4\mathbf{r}$.

WHERE THE MARKS ARE	
1 mark	$\mathbf{v}$
1 mark	Speed 6
1 mark	$\mathbf{a} = -4\mathbf{r}$

The path is the circle $x^2 + y^2 = 9$, and $\mathbf{a}$ points to its centre.

11

$\cos\theta = \frac{3}{5}$, $\sin\theta = \frac{4}{5}$, so $\mathbf{v}(0) = 15\mathbf{i} + 20\mathbf{j}$. Integrating, $\mathbf{v} = 15\mathbf{i} + (20 - 10t)\mathbf{j}$ and, with $\mathbf{r}(0) = \mathbf{0}$, $\mathbf{r} = 15t\,\mathbf{i} + (20t - 5t^2)\mathbf{j}$.

$5t(4 - t) = 0$: time of flight 4 s, range $15 \times 4 = 60$ m.

WHERE THE MARKS ARE	
1 mark	Derivation with constants
1 mark	Time of flight
1 mark	Range

Worked solutions · Advanced

12

(a) $\mathbf{u}\cdot\mathbf{v} = 2m + 2(m-1) - 4 = 4m - 6 = 0$, so $m = \frac{3}{2}$.

(b) Parallel needs $\mathbf{u} = \lambda\mathbf{v}$. The $\mathbf{k}$ components give $\lambda = -\frac{1}{4}$; then $m = 2\lambda = -\frac{1}{2}$, and the $\mathbf{j}$ components need $2 = \lambda(m-1) = -\frac{1}{4} \times \left(-\frac{3}{2}\right) = \frac{3}{8}$, which is false. So no value of m makes them parallel.

WHERE THE MARKS ARE	
1 mark	Scalar product set to 0
1 mark	$m = \frac{3}{2}$
1 mark	Contradiction shown for parallel

13

The upstream part of the swimmer's velocity must cancel the current: $1.5\sin\alpha = 0.9$, so $\sin\alpha = 0.6$ and $\alpha \approx 36.9°$ upstream of straight across.

Speed across $= \sqrt{1.5^2 - 0.9^2} = 1.2$ m/s, time $= \frac{40}{1.2} \approx 33.3$ s.

WHERE THE MARKS ARE	
1 mark	Diagram with 1.5 as the hypotenuse
1 mark	Angle upstream
1 mark	Time

14

$\mathbf{v} = 2t\,\mathbf{i} - 3t^2\,\mathbf{j} + 3\mathbf{j} = 2t\,\mathbf{i} + (3 - 3t^2)\mathbf{j}$; $\mathbf{r} = t^2\mathbf{i} + (3t - t^3)\mathbf{j} + \mathbf{i}$, so $\mathbf{r} = (t^2 + 1)\mathbf{i} + (3t - t^3)\mathbf{j}$.

$3t - t^3 = t(3 - t^2) = 0$: next at $t = \sqrt{3}$. Then $\mathbf{v} = 2\sqrt{3}\,\mathbf{i} - 6\mathbf{j}$, speed $\sqrt{12 + 36} = 4\sqrt{3}$.

WHERE THE MARKS ARE	
1 mark	$\mathbf{v}$ with constant
1 mark	$\mathbf{r}$ with constant
1 mark	$4\sqrt{3}$ at $t = \sqrt{3}$

15

$\overrightarrow{AB} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\overrightarrow{AC} = \mathbf{i} + 4\mathbf{j} + \mathbf{k}$, and $\overrightarrow{AB} \cdot \overrightarrow{AC} = 2 - 4 + 2 = 0$: $\angle BAC = 90°$.

$|\overrightarrow{AB}| = 3$, $|\overrightarrow{AC}| = \sqrt{18} = 3\sqrt{2}$. Area $= \frac{1}{2} \times 3 \times 3\sqrt{2} = \frac{9\sqrt{2}}{2}$ square units.

WHERE THE MARKS ARE	
1 mark	Both vectors from A
1 mark	Dot product 0
1 mark	Area

Worked solutions · Exam-style

16 (a)

$\mathbf{v}(0) = V\cos 45°\mathbf{i} + V\sin 45°\mathbf{j} = \frac{V}{\sqrt{2}}\mathbf{i} + \frac{V}{\sqrt{2}}\mathbf{j}$, $\mathbf{r}(0) = 15\mathbf{j}$.

$$\mathbf{v} = -10t\,\mathbf{j} + \mathbf{c}_1 = \frac{V}{\sqrt{2}}\mathbf{i} + \left(\frac{V}{\sqrt{2}} - 10t\right)\mathbf{j} \quad \text{t = 0 gives } \mathbf{c}_1$$

$$\mathbf{r} = \frac{Vt}{\sqrt{2}}\mathbf{i} + \left(\frac{Vt}{\sqrt{2}} - 5t^2\right)\mathbf{j} + \mathbf{c}_2,\ \mathbf{c}_2 = 15\mathbf{j} \quad \text{t = 0 gives } \mathbf{c}_2$$

So $\mathbf{r}(t) = \frac{Vt}{\sqrt{2}}\mathbf{i} + \left(15 + \frac{Vt}{\sqrt{2}} - 5t^2\right)\mathbf{j}$, as required.

MARKING GUIDELINES	
CRITERIA	MARKS
Provides a correct derivation	2
Finds $\mathbf{v}$ correctly, or equivalent merit	1

16 (b)

It lands when $x = 45$ and $y = 0$. From x: $\frac{Vt}{\sqrt{2}} = 45$. Substituting into y: $15 + 45 - 5t^2 = 0$, so $t^2 = 12$ and $t = 2\sqrt{3}$ (positive).

$V = \frac{45\sqrt{2}}{2\sqrt{3}} = \frac{45\sqrt{6}}{6} = \boxed{\frac{15\sqrt{6}}{2}\text{ m/s}}$, about 18.4 m/s.

MARKING GUIDELINES

CRITERIA	MARKS
Provides the correct value of V	3
Finds the time of flight, or equivalent merit	2
Uses $x = 45$ and $y = 0$ at the same time, or equivalent merit	1

17

(a) $\mathbf{a} \cdot \mathbf{b} = 6 + 0 - 8 = \boxed{-2}$, $|\mathbf{a}| = \boxed{3}$, $|\mathbf{b}| = \boxed{5}$.

(b) $\cos\theta = -\frac{2}{15}$, $\boxed{\theta \approx 97.7^\circ}$ (obtuse, as the negative dot product says).

(c) $\text{proj}_{\mathbf{b}}\,\mathbf{a} = -\frac{2}{25}(3\mathbf{i} + 4\mathbf{k}) = \boxed{-\frac{6}{25}\mathbf{i} - \frac{8}{25}\mathbf{k}}$, pointing against $\mathbf{b}$.

(d) $\mathbf{a} - \text{proj}_{\mathbf{b}}\,\mathbf{a} = \boxed{\frac{56}{25}\mathbf{i} + \mathbf{j} - \frac{42}{25}\mathbf{k}}$. Check: $\frac{56}{25} \times 3 + 0 - \frac{42}{25} \times 4 = \frac{168-168}{25} = 0$, so it is perpendicular to $\mathbf{b}$.

MARKING GUIDELINES

CRITERIA	MARKS
(a) Correct values	1
(b) Correct angle	1
(c) Correct projection	1
(d) Correct perpendicular component with the verification	2
(d) Correct perpendicular component, or equivalent merit	1

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