MA99 EDUCATION

Mathematics Advanced: Practice Paper 1

01

NSW MATHEMATICS ADVANCED 11-12 SYLLABUS 2024

FOR HSC 2028 AND LATER

General Instructions

- Reading time: 10 minutes
- Working time: 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- The NESA Mathematics Advanced reference sheet may be used. In an examination, use the current official NESA sheet.
- In Questions 11 onwards, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I · 10 marks

Questions 1 to 10

Suggested time: about 15 minutes

Section II · 60 marks

Questions 11 to 14

Suggested time: about 1 hour 45 minutes

This paper examines the whole Year 11 Mathematics Advanced course of the 2024 syllabus: functions, trigonometric functions, calculus, exponential and logarithmic functions, and statistical analysis (probability and data), with the most marks on functions, trigonometric functions and calculus. Questions with a setting are drawn from the coast and surf lifesaving; all data and models in them are hypothetical (MA99).

Marks may be deducted for careless or badly arranged work.

Section I · Answer Sheet

Shade ONE circle for each of Questions 1 to 10. If you change your mind, draw a cross through the incorrect answer and shade the new one.

1	A	B	C	D	2	A	B	C	D
3	A	B	C	D	4	A	B	C	D
5	A	B	C	D	6	A	B	C	D
7	A	B	C	D	8	A	B	C	D
9	A	B	C	D	10	A	B	C	D

Section I · 10 marks

Answer Questions 1 to 10 on the multiple choice answer sheet, spending about 15 minutes on this section.

Question 1

1 MARK

For which values of k does the equation $kx^2 - 6x + 3 = 0$ have no real roots?

A $k < 3$
B $k > 3$
C $k > -3$
D $k \geq 3$

Question 2

1 MARK

The functions f and g are given by $f(x) = 3 - 2x$ and $g(x) = x^2 - 1$. What is the value of $f(g(-2))$?

A 48
B 13
C 21
D -3

Question 3

1 MARK

The point $(4, -2)$ lies on the graph of $y = f(x)$. Which point must lie on the graph of $y = 3f\left(\frac{x}{2}\right) - 1$?

A $(8, -7)$
B $(2, -7)$
C $(8, -9)$
D $(4, -7)$

Question 4

1 MARK

A sector of a circle has angle $\frac{\pi}{3}$ radians and area 12π square units. What is the radius of the circle?

A 6
B 72
C $6\sqrt{2}$
D $3\sqrt{2}$

Question 5

1 MARK

Given that $\tan\theta = -\frac{3}{4}$ and $90^\circ < \theta < 180^\circ$, what is the value of $\operatorname{cosec}\theta$?

A $-\frac{5}{3}$
B $\frac{5}{3}$
C $\frac{3}{5}$
D $\frac{5}{4}$

Question 6

1 MARK

What is the gradient of the curve $y = \dfrac{x^2 + 1}{2x - 3}$ at the point where $x = 3$?

A $-\dfrac{2}{9}$

B $\dfrac{2}{9}$

C $-\dfrac{2}{3}$

D 3

Question 7

1 MARK

What is the equation of the normal to the curve $y = x^3 - 3x^2 + 2$ at the point where $x = 3$?

A $9x - y - 25 = 0$

B $9x + y - 29 = 0$

C $x - 9y + 15 = 0$

D $x + 9y - 21 = 0$

Question 8

1 MARK

For which values of x is the function $f(x) = 2x^3 - 3x^2 - 12x + 5$ decreasing?

A $x < 2$

B $x < -1$ or $x > 2$

C $-1 < x < 2$

D $-2 < x < 1$

Question 9

1 MARK

Given that $x = \log_3 5$, which of the following is equal to $\log_9 125$?

A $\dfrac{3x}{2}$

B $3x$

C $\dfrac{2x}{3}$

D $6x$

Question 10

1 MARK

For two events A and B from the same sample space, $P(A) = 0.6$, $P(B) = 0.4$ and $P(A \cup B) = 0.7$. What is the value of $P(A \mid B)$?

A $\dfrac{1}{2}$

B $\dfrac{3}{10}$

C $\dfrac{3}{7}$

D $\dfrac{3}{4}$

Section II · 60 marks

Answer Questions 11 to 14 in the spaces provided, spending about 1 hour 45 minutes on this section. Show your mathematical reasoning and/or calculations in every answer.

Question 11 · 15 marks

(a) Simplify $\dfrac{1}{x-2} - \dfrac{4}{x^2-4}$. 2 MARKS

(b) Express $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ in the form $a+\sqrt{b}$, where a and b are integers. 2 MARKS

(c) The time T minutes that a surf rescue boat takes to reach a swimmer over a fixed course varies inversely with the boat's average speed v km/h. The relationship is hypothetical (MA99).

(i) Given that $T = 9$ when $v = 20$, find the value of T when $v = 30$. 2 MARKS

(ii) The boat must reach a swimmer in less than 5 minutes. Find the values of v for which it does. 2 MARKS

(d) Find the exact value of $\cos(-210^\circ) + \tan 225^\circ$. 2 MARKS

(e) Solve $2\cos^2 x + \sin x = 1$ for $0 \leq x \leq 2\pi$. 3 MARKS

(f) A surf club has 50 members. Of these, 31 volunteer for beach patrols, 24 coach junior nippers and 9 do neither (hypothetical, MA99). Find the number of members who do both, and hence find the probability that a member chosen at random volunteers for beach patrols only. 2 MARKS

Question 12 · 15 marks

(a) Use differentiation from first principles to show that the derivative of $f(x) = 2x^2 - 3x$ is $f'(x) = 4x - 3$. 3 MARKS

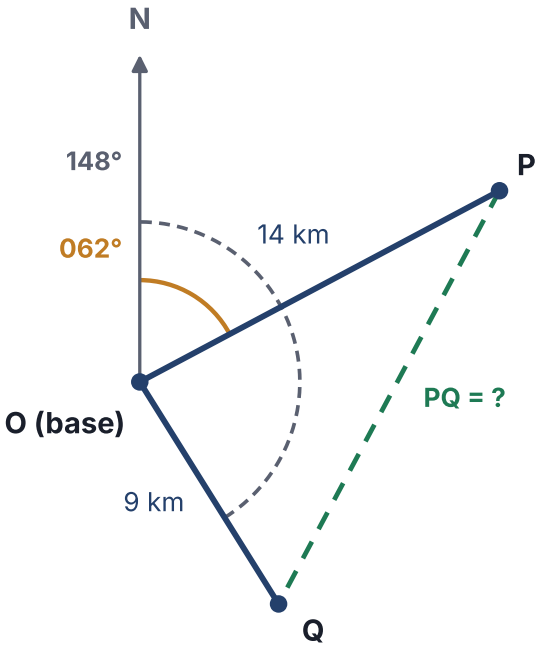
(b) Differentiate:

(i) $y = (x^2 + 3)^5$ 1 MARK

(ii) $y = \dfrac{x}{x^2 + 1}$, simplifying your answer. 2 MARKS

(c) Let $f(x) = 2x^3 - 9x^2 + 12x - 3$. Find the coordinates of the stationary points of the graph of $y = f(x)$ and determine their nature. 4 MARKS

(d) Two rescue boats leave a base O at the same time. Boat P travels 14 km on a bearing of 062° and boat Q travels 9 km on a bearing of 148° (hypothetical, MA99).



Question 12 (d) · the paths of the two boats (not to scale)

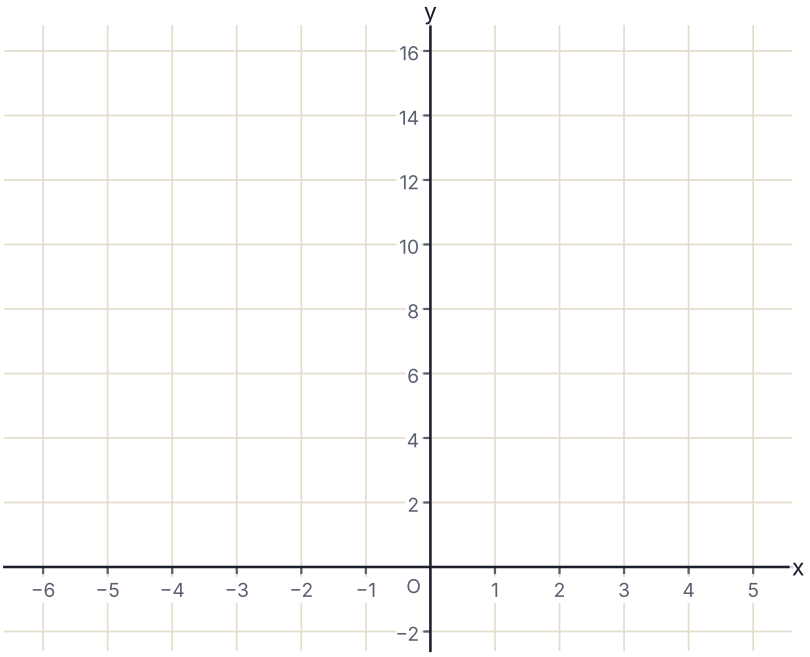
(i) Find the distance PQ, correct to 1 decimal place. 2 MARKS

(ii) Find the bearing of Q from P, to the nearest degree. 2 MARKS

(e) The time, in minutes, that a lifeguard takes to reach a swimmer is recorded on many rescues. State whether this random variable is discrete or continuous, and give a reason. 1 MARK

Question 13 · 15 marks

(a) The axes below are provided for parts (i) and (ii).



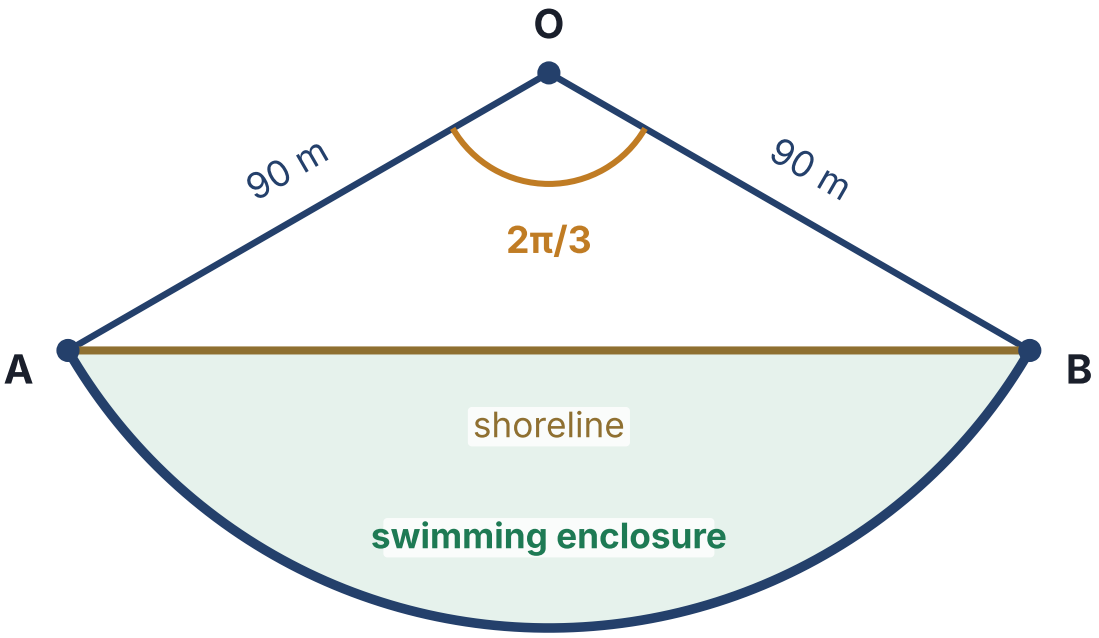
Question 13 (a) · axes for the sketch

(i) On the axes, sketch the graphs of $y = |3x + 6|$ and $y = 2x + 9$, showing the intercepts with the axes. 2 MARKS

(ii) Solve $|3x + 6| = 2x + 9$, and check your solutions against your graph. 2 MARKS

(b) A circular swimming zone is marked on a map grid with units in metres. Its boundary has equation $x^2 + y^2 - 10x + 2y - 23 = 0$. Find the centre and radius of the zone, and show that a flag at the point $(5, 6)$ lies on its boundary. 3 MARKS

(c) A swimming enclosure is bounded by the straight shoreline AB and the arc AB of a circle with centre O and radius 90 m, where $\angle AOB = \dfrac{2\pi}{3}$ (hypothetical, MA99). Find the exact area of the enclosure. 3 MARKS



Question 13 (c) · the enclosure (not to scale)

(d) A rescue craft travels along a straight line from a boat ramp. Its displacement x kilometres from the ramp after t hours is $x = t^3 - 6t^2 + 9t$, for $0 \leq t \leq 4$ (hypothetical model, MA99).

(i) Find an expression for the velocity of the craft at time t. 1 MARK

(ii) Find the times at which the craft is at rest. 2 MARKS

(iii) Find the total distance the craft travels in the first 4 hours. 2 MARKS

Question 14 · 15 marks

(a) A harmful algal bloom is modelled by $A = 120e^{0.08t}$, where A is the number of cells per millilitre of water and t is the time in days since the first count (hypothetical model, MA99).

(i) Find the initial count, and the count after 10 days to the nearest whole number. 2 MARKS

(ii) After how many days does the count first reach 1000 cells per millilitre? Give your answer correct to 1 decimal place. 2 MARKS

(b) Let $x > 0$.

(i) Use the change of base rule to show that $\log_9 x = \dfrac{1}{2}\log_3 x$. 1 MARK

(ii) Hence, or otherwise, solve $2\log_9 x + \log_3(x-2) = 2$. 3 MARKS

(c) Prove that $\tan x + \cot x = \sec x \operatorname{cosec} x$, for all x for which both sides are defined. 3 MARKS

(d) On a summer day, the probability that a southerly change arrives at a beach is 0.25. If a southerly change arrives, the probability that the surf is rough is 0.4. If it does not, the probability that the surf is rough is 0.1 (hypothetical (MA99)).

(i) Find the probability that the surf is rough. 2 MARKS

(ii) The surf is rough. Find the probability that a southerly change arrived. 2 MARKS

End of paper.

About this resource

MA99 EDUCATION

Scope

NSW Mathematics Advanced, Year 11. Written to the NSW Mathematics Advanced 11-12 Syllabus (2024), all Year 11 focus areas, for students sitting the HSC in 2028 and later years. Every value and solution was re-derived independently by computer algebra before printing. Contexts in the questions are invented unless a source is named.

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MA99 EDUCATION

Mathematics Advanced: Practice Paper 1 · Worked Solutions

01

NSW MATHEMATICS ADVANCED 11-12 SYLLABUS 2024 FOR HSC 2028 AND LATER

Worked solutions · Section I

SECTION I KEY: 1 B · 2 D · 3 A · 4 C · 5 B · 6 A · 7 D · 8 C · 9 A · 10 D

1

The equation $kx^2 - 6x + 3 = 0$ has no real roots when its discriminant is negative:

$$\Delta = (-6)^2 - 4(k)(3) = 36 - 12k < 0 \implies k > 3$$

(B)

(A) is the condition $\Delta > 0$ for two distinct real roots: dividing $36 - 12k < 0$ by -12 without reversing the inequality gives $k < 3$. (C) writes $(-6)^2$ as -36, so $-36 - 12k < 0$ gives $k > -3$. (D) includes $k = 3$, where $\Delta = 0$ and the equation has one repeated real root; no real roots needs $\Delta < 0$ strictly.

2

Work from the inside out. $g(-2) = (-2)^2 - 1 = 3$, so $f(g(-2)) = f(3) = 3 - 2(3) = -3$.

(D)

(A) is $g(f(-2))$, the composite taken the wrong way round: $f(-2) = 7$ and $g(7) = 48$. (B) reads $(-2)^2$ as -4: $g(-2) = -5$ and $f(-5) = 13$. (C) multiplies $f(-2) = 7$ by $g(-2) = 3$ instead of putting one function inside the other.

3

In $y = 3f\left(\frac{x}{2}\right) - 1$ the factor $\frac{x}{2}$ stretches horizontally by a factor of 2, the 3 stretches vertically by a factor of 3, and the -1 is applied last. A point (a, b) on $y = f(x)$ moves to $(2a,\ 3b - 1)$. For $(4, -2)$ that is $(8,\ 3(-2) - 1) = (8, -7)$.

(A)

(B) halves the x-coordinate; replacing x by $\frac{x}{2}$ doubles it, because $\frac{x}{2} = 4$ at $x = 8$. (C) subtracts 1 before the vertical stretch: $3(-2 - 1) = -9$ instead of $3(-2) - 1$. (D) applies the vertical changes but ignores the horizontal dilation.

4

The area of a sector is $A = \frac{1}{2}r^2\theta$, so $12\pi = \frac{1}{2}r^2\left(\frac{\pi}{3}\right)$:

$$r^2 = \frac{2 \times 12\pi}{\pi/3} = 72$$

$$r = \sqrt{72} = 6\sqrt{2} \quad r > 0$$

(C)

(A) uses $A = r^2\theta$ and leaves out the $\frac{1}{2}$: $r^2 = 36$. (B) stops at $r^2 = 72$ and forgets to take the square root. (D) isolates r^2 as $\frac{A}{2\theta} = 18$ instead of $\frac{2A}{\theta} = 72$.

5

$\tan\theta = -\dfrac{3}{4}$ with $90^\circ < \theta < 180^\circ$: in the second quadrant $y > 0$ and $x < 0$, so take $y = 3$, $x = -4$ and $r = \sqrt{9+16} = 5$. Then $\sin\theta = \dfrac{3}{5}$ and

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{5}{3}$$

(B)

(A) gives sine the sign of the tangent; sine is positive in the second quadrant. (C) is $\sin\theta$ itself, not its reciprocal. (D) uses the adjacent side 4 instead of the opposite side 3; it is $\left|\sec\theta\right|$.

6

Quotient rule with $u = x^2 + 1$ and $v = 2x - 3$, so $u' = 2x$ and $v' = 2$:

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2} = \frac{(2x-3)(2x) - (x^2+1)(2)}{(2x-3)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=3} = \frac{(3)(6) - (10)(2)}{3^2} = \frac{-2}{9}$$

(A)

(B) reverses the numerator of the quotient rule, $uv' - vu'$ instead of $vu' - uv'$. (C) forgets to square the denominator: $\dfrac{-2}{3}$ instead of $\dfrac{-2}{3^2}$. (D) differentiates the numerator and the denominator separately and divides: $\dfrac{2x}{2} = x$.

7

At $x = 3$: $y = 27 - 27 + 2 = 2$. The derivative is $y' = 3x^2 - 6x$, so the tangent gradient is $y'(3) = 9$ and the normal gradient is $-\dfrac{1}{9}$. The normal is

$$y - 2 = -\tfrac{1}{9}(x - 3)$$

$$9y - 18 = -x + 3 \implies x + 9y - 21 = 0$$

(D)

(A) is the tangent at $x = 3$, not the normal. (B) uses the gradient -9, the negative of the tangent gradient, instead of its negative reciprocal. (C) uses the reciprocal $\dfrac{1}{9}$ and forgets the negative sign.

8

$f'(x) = 6x^2 - 6x - 12 = 6(x-2)(x+1)$. The function is decreasing where $f'(x) < 0$. The upward parabola $y = f'(x)$ lies below the x-axis between its zeroes -1 and 2, so $-1 < x < 2$.

(C)

(A) uses only the factor $x - 2 < 0$ and ignores the factor $x + 1$. (B) is where $f'(x) > 0$, so f is increasing, the outer regions of the upward parabola. (D) comes from a sign error in the derivative, $6x^2 + 6x - 12$, whose zeroes are -2 and 1.

9

Change base to 3: $\log_9 125 = \dfrac{\log_3 125}{\log_3 9} = \dfrac{3\log_3 5}{2} = \dfrac{3x}{2}$.

(A)

(B) writes $\log_9 125 = 3\log_9 5$ and then treats $\log_9 5$ as x, ignoring that the base is 9. (C) is the reciprocal of the correct multiple of x. (D) multiplies $3x$ by $\log_3 9 = 2$ instead of dividing by it.

10

From $P(A \cup B) = P(A) + P(B) - P(A \cap B)$: $0.7 = 0.6 + 0.4 - P(A \cap B)$, so $P(A \cap B) = 0.3$. Then

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.4} = \frac{3}{4}$$

(D)

(A) is $P(B \mid A) = \dfrac{0.3}{0.6}$, the conditional probability the wrong way round. (B) is $P(A \cap B)$ itself, not divided by $P(B)$. (C) divides by $P(A \cup B) = 0.7$ instead of $P(B) = 0.4$.

Using the Section I solutions

For each item you missed, find the distractor you chose above: it names the exact error that produces it. Write that error down before you look at Section II.

Worked solutions · Section II

Question 11

11 (a)

Factorise $x^2 - 4 = (x-2)(x+2)$ and use it as the common denominator, with $x \neq \pm 2$:

$$\frac{1}{x-2} - \frac{4}{x^2-4} = \frac{x+2}{(x-2)(x+2)} - \frac{4}{(x-2)(x+2)}$$

$$= \frac{x-2}{(x-2)(x+2)} = \frac{1}{x+2} \quad \text{cancel } x-2$$

$$\frac{1}{x+2}$$

MARKING GUIDELINES	
CRITERIA	MARKS
Provides correct solution	2
Writes both fractions over the common denominator $(x-2)(x+2)$, or equivalent merit	1

Factorise first. Multiplying the two denominators gives a cubic that hides the cancellation of $x - 2$.

11 (b)

Multiply the numerator and the denominator by the conjugate $\sqrt{5} + \sqrt{3}$ of the denominator:

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{5 + 2\sqrt{15} + 3}{5-3}$$

$$= \frac{8 + 2\sqrt{15}}{2} = 4 + \sqrt{15}$$

$4 + \sqrt{15}$, so $a = 4$ and $b = 15$.

MARKING GUIDELINES	
CRITERIA	MARKS
Provides correct solution	2
Multiplies by the correct conjugate and obtains a denominator of 2, or equivalent merit	1

11 (c) (i)

Inverse variation means $T = \dfrac{k}{v}$. With $T = 9$ and $v = 20$: $k = 9 \times 20 = 180$, so $T = \dfrac{180}{v}$.

When $v = 30$: $T = \dfrac{180}{30} =$ 6 minutes.

MARKING GUIDELINES	
CRITERIA	MARKS
Provides correct solution	2
Finds $k = 180$, or equivalent merit	1

11 (c) (ii)

$T < 5$ means $\dfrac{180}{v} < 5$. Since $v > 0$, multiplying by v does not reverse the inequality:

$$180 < 5v$$

$$v > 36$$

The boat must average more than 36 km/h.

MARKING GUIDELINES	
CRITERIA	MARKS
Provides correct solution	2
Writes $\dfrac{180}{v} < 5$ or $180 < 5v$, or equivalent merit	1

A longer trip time needs a lower speed. Students who write $v < 36$ have reversed the inequality without a reason: v is positive, so the inequality keeps its direction.

11 (d)

Cosine is even, so $\cos(-210°) = \cos 210°$. The related angle is $30°$ and cosine is negative in the third quadrant, so $\cos 210° = -\dfrac{\sqrt{3}}{2}$. Similarly $225° = 180° + 45°$ and tangent is positive in the third quadrant, so $\tan 225° = \tan 45° = 1$.

$$\cos(-210°) + \tan 225° = -\frac{\sqrt{3}}{2} + 1 = \frac{2 - \sqrt{3}}{2}$$

$1 - \dfrac{\sqrt{3}}{2}$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Finds one of the two exact values correctly, or equivalent merit	1

11 (e)

Replace $\cos^2 x$ using $\cos^2 x = 1 - \sin^2 x$, so that the equation involves only $\sin x$:

$$2(1 - \sin^2 x) + \sin x = 1$$

$$2\sin^2 x - \sin x - 1 = 0 \quad \text{rearrange}$$

$$(2\sin x + 1)(\sin x - 1) = 0 \quad \text{factorise}$$

$$\sin x = -\tfrac{1}{2} \quad \text{or} \quad \sin x = 1$$

For $\sin x = 1$: $x = \dfrac{\pi}{2}$. For $\sin x = -\dfrac{1}{2}$ the related angle is $\dfrac{\pi}{6}$ and sine is negative in the third and fourth quadrants: $x = \pi + \dfrac{\pi}{6}$ and $x = 2\pi - \dfrac{\pi}{6}$.

$$x = \frac{\pi}{2},\ \frac{7\pi}{6},\ \frac{11\pi}{6}$$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Obtains $(2\sin x + 1)(\sin x - 1) = 0$ and solves for $\sin x$, or finds the solutions from one correct value of $\sin x$, or equivalent merit	2
Uses $\cos^2 x = 1 - \sin^2 x$ to form a quadratic in $\sin x$, or equivalent merit	1

Check the interval: $\sin x = -\dfrac{1}{2}$ gives two angles in $[0, 2\pi]$, and $\sin x = 1$ gives one. Forgetting the third and fourth quadrant pair costs the last mark.

11 (f)

Let P be the set of members who patrol and C the set who coach. Then $|P \cup C| = 50 - 9 = 41$.

$$|P \cap C| = |P| + |C| - |P \cup C| = 31 + 24 - 41 = 14$$

Patrol only: $31 - 14 = 17$ members, so the probability is $\dfrac{17}{50}$ (and 14 members do both).

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Finds that 14 members do both, or equivalent merit	1

Question 12

12 (a)

$$f(x+h)-f(x)=2(x+h)^2-3(x+h)-2x^2+3x=4xh+2h^2-3h \quad \text{expand and cancel}$$

$$\frac{f(x+h)-f(x)}{h}=\frac{4xh+2h^2-3h}{h}=4x+2h-3 \quad \text{divide by } h,\ h\neq 0$$

$$f'(x)=\lim_{h\to 0}(4x+2h-3)=4x-3$$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Simplifies the difference quotient to $4x+2h-3$, or equivalent merit	2
Correctly writes the difference quotient and expands $f(x+h)$, or equivalent merit	1

The last line needs the limit notation: $f'(x)=\lim_{h\to 0}$ of the simplified quotient. Setting $h=0$ before dividing by h gives $\frac{0}{0}$.

12 (b) (i)

Chain rule with $u=x^2+3$: $\frac{dy}{dx}=5u^4\times 2x=$ $10x(x^2+3)^4$.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct answer	1

12 (b) (ii)

Quotient rule with $u=x$ and $v=x^2+1$, so $u'=1$ and $v'=2x$:

$$\frac{dy}{dx}=\frac{(x^2+1)(1)-x(2x)}{(x^2+1)^2}=\frac{1-x^2}{(x^2+1)^2}$$

$$\frac{1-x^2}{(x^2+1)^2}$$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Correctly applies the quotient rule, or equivalent merit	1

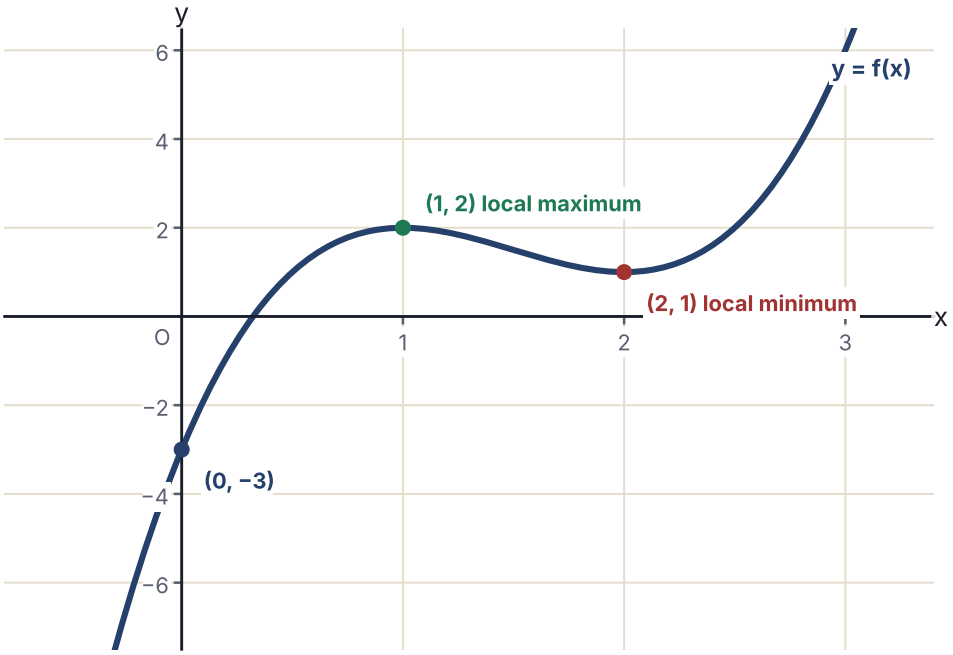
12 (c)

$f'(x)=6x^2-18x+12=6(x-1)(x-2)$. Stationary points are where $f'(x)=0$: $x=1$ and $x=2$.

$f(1)=2-9+12-3=2$ and $f(2)=16-36+24-3=1$. Test the sign of $f'(x)$ on each side:

x	0	1	1.5	2	3
$f'(x)$	12	0	-1.5	0	12
Gradient	positive	stationary	negative	stationary	positive

The gradient changes from positive to negative at $x = 1$ and from negative to positive at $x = 2$.



Question 12 (c) · the stationary points of $y = 2x^3 - 9x^2 + 12x - 3$

$(1, 2)$ is a local maximum and $(2, 1)$ is a local minimum.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	4
Finds both stationary points and justifies the nature of one, or equivalent merit	3
Finds the x-values of both stationary points and one y-value, or equivalent merit	2
Finds $f'(x)$ and sets it to 0, or equivalent merit	1

A stationary point is a maximum or a minimum only once the sign of $f'(x)$ on each side has been shown. Stating $f'(1) = 0$ alone earns no justification mark.

12 (d) (i)

The bearings differ by $148^\circ - 062^\circ = 86^\circ$, so $\angle POQ = 86^\circ$. By the cosine rule in triangle POQ:

$$PQ^2 = 14^2 + 9^2 - 2(14)(9)\cos 86^\circ = 277 - 252\cos 86^\circ \approx 259.42$$

$$PQ \approx 16.1$$

$PQ \approx 16.1$ km

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Uses $\angle POQ = 86^\circ$ in the cosine rule, or equivalent merit	1

Work in degree mode. The angle between the two bearings is found by subtracting them: it is not $62^\circ + 148^\circ$.

12 (d) (ii)

By the sine rule in triangle POQ (with $PQ = 16.1066\ldots$): $\dfrac{\sin \angle OPQ}{9} = \dfrac{\sin 86^\circ}{PQ}$, so $\sin \angle OPQ \approx 0.5574$. The angle opposite the shorter side OQ is acute: $\angle OPQ \approx 33.9^\circ$.

The bearing of O from P is $062^\circ + 180^\circ = 242^\circ$. Q lies on the anticlockwise side of the line PO as seen from P, so the bearing of Q from P is

$$242^\circ - 33.9^\circ \approx 208.1^\circ$$

The bearing of Q from P is 208°.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Finds $\angle OPQ \approx 33.9^\circ$, or the bearing of O from P as 242°, or equivalent merit	1

Check with a sketch: Q is south and a little west of P, so the bearing is between 180° and 270°, near 208° and not $242^\circ + 33.9^\circ$.

12 (e)

Continuous: the time can take any value in an interval, including fractions of a minute, so it is measured rather than counted.

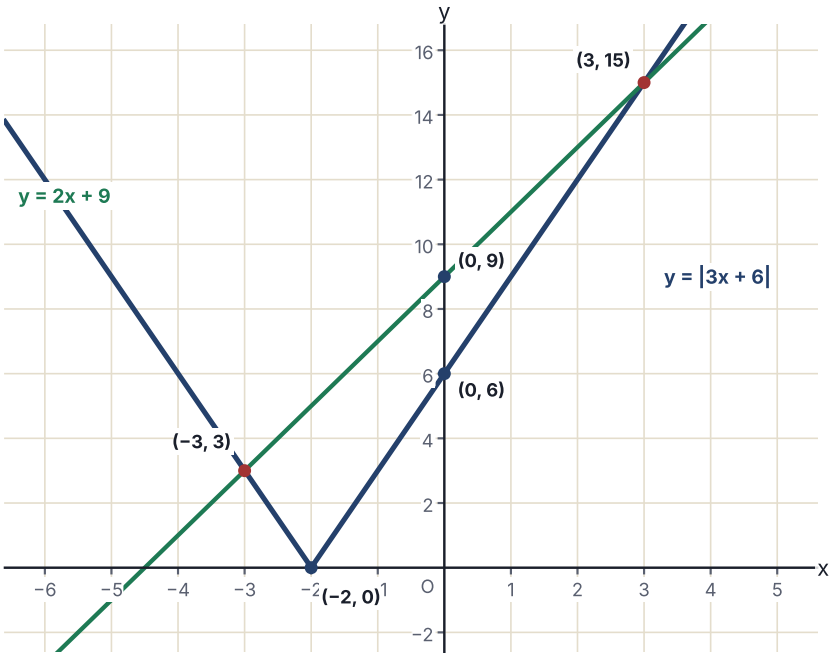
MARKING GUIDELINES

CRITERIA	MARKS
Provides correct answer with a reason	1

Question 13

13 (a) (i)

$y = |3x + 6|$ has its vertex where $3x + 6 = 0$, at $(-2, 0)$, and y-intercept 6. The line $y = 2x + 9$ has y-intercept 9 and x-intercept -4.5.



Question 13 (a) · the graph of $y = |3x + 6|$ meets the line $y = 2x + 9$ twice

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct sketch of both graphs with intercepts	2
Correctly sketches one graph with its intercepts, or both graphs with errors in the intercepts	1

13 (a) (ii)

Split into the two cases for the absolute value:

$$3x + 6 = 2x + 9 \implies x = 3 \quad 3x + 6 \geq 0$$

$$-(3x + 6) = 2x + 9 \implies -5x = 15 \implies x = -3 \quad 3x + 6 < 0$$

Check: at $x = 3$, $|15| = 15 = 2(3) + 9$; at $x = -3$, $|-3| = 3 = 2(-3) + 9$. Both agree with the points $(3, 15)$ and $(-3, 3)$ on the graph. $x = 3$ or $x = -3$.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution with a check	2
Solves one case correctly, or equivalent merit	1

An absolute value equation with x on both sides must be checked: the right-hand side $2x + 9$ has to be non-negative for any solution. Both solutions pass here.

13 (b)

Complete the square in x and in y:

$$x^2 - 10x + y^2 + 2y = 23$$

$$(x-5)^2 - 25 + (y+1)^2 - 1 = 23$$

$$(x-5)^2 + (y+1)^2 = 49$$

The centre is $(5, -1)$ and the radius is 7 m. At the flag $(5, 6)$: $(5-5)^2 + (6+1)^2 = 0 + 49 = 49$, so the flag lies on the boundary.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Finds the centre and radius, or equivalent merit	2
Completes the square for x or for y, or equivalent merit	1

13 (c)

Area of the enclosure = area of sector AOB minus area of triangle AOB, with $\theta = \dfrac{2\pi}{3}$ and $r = 90$:

$$\text{sector} = \tfrac{1}{2}r^2\theta = \tfrac{1}{2}(90)^2\left(\tfrac{2\pi}{3}\right) = 2700\pi$$

$$\text{triangle} = \tfrac{1}{2}r^2\sin\theta = \tfrac{1}{2}(90)^2\left(\tfrac{\sqrt{3}}{2}\right) = 2025\sqrt{3}$$

$$\text{segment} = 2700\pi - 2025\sqrt{3}$$

$\left(2700\pi - 2025\sqrt{3}\right)$ m^2, about 4975 m^2.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Finds the area of the sector and the area of the triangle, or equivalent merit	2
Finds the area of the sector or the area of the triangle, or equivalent merit	1

The triangle area uses $\dfrac{1}{2}ab\sin C$ with $\sin\dfrac{2\pi}{3} = \dfrac{\sqrt{3}}{2}$. Using $\dfrac{1}{2}bh$ with a guessed height, or putting θ in degrees in $\dfrac{1}{2}r^2\theta$, are the usual losses.

13 (d) (i)

$v = \dfrac{dx}{dt} = 3t^2 - 12t + 9$ km/h.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct answer	1

13 (d) (ii)

At rest means $v = 0$: $3t^2 - 12t + 9 = 3(t-1)(t-3) = 0$.

The craft is at rest when $t = 1$ and $t = 3$ hours.

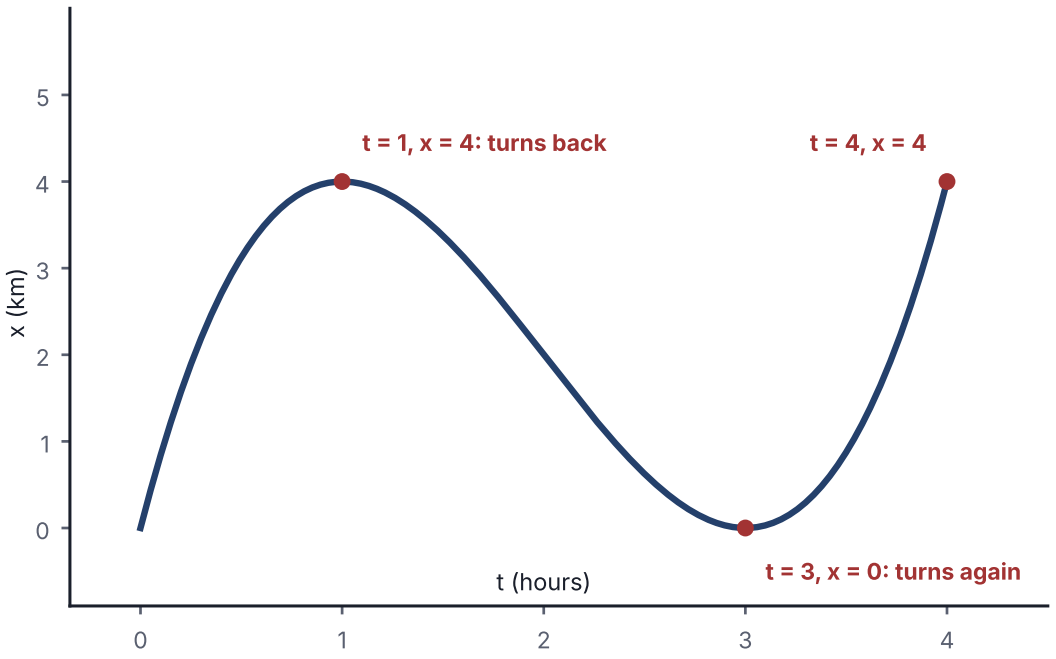
MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Sets $v = 0$ and factorises or solves, or equivalent merit	1

13 (d) (iii)

The craft turns round at $t = 1$ and $t = 3$, so split the motion there. Displacements: $x(0) = 0$, $x(1) = 4$, $x(3) = 0$, $x(4) = 4$.

Interval	From x	To x	Distance (km)
$0 \le t \le 1$	0	4	4
$1 \le t \le 3$	4	0	4
$3 \le t \le 4$	0	4	4



Question 13 (d) · the craft turns back at $t = 1$ and $t = 3$

Total distance $= 4 + 4 + 4 =$ 12 km. The displacement after 4 hours is only 4 km.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Uses the turning points to split the motion, or finds the displacement at $t = 0, 1, 3, 4$, or equivalent merit	1

Distance is not displacement. Computing $x(4) - x(0) = 4$ gives the displacement and loses both marks.

Question 14

14 (a) (i)

At $t = 0$: $A = 120e^{0} = \boxed{120}$ cells per mL. At $t = 10$: $A = 120e^{0.8} \approx \boxed{267}$ cells per mL.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Finds one value correctly, or equivalent merit	1

14 (a) (ii)

$$120e^{0.08t} = 1000$$

$$e^{0.08t} = \frac{25}{3}$$

$$0.08t = \ln \frac{25}{3} \approx 2.1203 \quad \text{take natural logarithms}$$

$$t \approx 26.5$$

$\boxed{\text{About } 26.5 \text{ days}}$.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Obtains $e^{0.08t} = \frac{25}{3}$ or $0.08t = \ln \frac{25}{3}$, or equivalent merit	1

14 (b) (i)

By the change of base rule with base 3: $\log_9 x = \dfrac{\log_3 x}{\log_3 9} = \dfrac{\log_3 x}{2}$, because $3^2 = 9$. Hence $\log_9 x = \dfrac{1}{2}\log_3 x$.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	1

14 (b) (ii)

$$2\log_9 x + \log_3(x-2) = 2$$

$\log_3 x + \log_3(x-2) = 2$ by part (i), $2\log_9 x = \log_3 x$

$\log_3 [x(x-2)] = 2$ log law

$x(x-2) = 3^2 = 9 \implies x^2 - 2x - 9 = 0$

$$x = \frac{2 \pm \sqrt{4+36}}{2} = 1 \pm \sqrt{10}$$

The logarithms need $x > 2$. Since $1 - \sqrt{10} < 0$, reject it. $x = 1 + \sqrt{10}$, which is about 4.16 and exceeds 2.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Obtains $x^2 - 2x - 9 = 0$ and solves it, or equivalent merit	2
Combines the logarithms into a single logarithm, or equivalent merit	1

Always check roots against the domain: $\log_3(x-2)$ needs $x > 2$, which removes $1 - \sqrt{10}$.

14 (c)

$$\text{LHS} = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \quad \tan x = \frac{\sin x}{\cos x}, \cot x = \frac{\cos x}{\sin x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \quad \text{common denominator}$$

$$= \frac{1}{\sin x \cos x} \quad \sin^2 x + \cos^2 x = 1$$

$$= \frac{1}{\cos x} \times \frac{1}{\sin x} = \sec x \operatorname{cosec} x = \text{RHS}$$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct proof	3
Writes the left-hand side as a single fraction in $\sin x$ and $\cos x$, or equivalent merit	2
Rewrites $\tan x$ and $\cot x$ in terms of $\sin x$ and $\cos x$, or equivalent merit	1

Start from the more complicated side and write everything in $\sin x$ and $\cos x$. Both sides are defined only when $\sin x \neq 0$ and $\cos x \neq 0$.

14 (d) (i)

Let S be 'a southerly change arrives' and R be 'the surf is rough'. $P(S) = 0.25$, $P(R \mid S) = 0.4$, $P(R \mid \overline{S}) = 0.1$. Adding the two branches of the tree that end in R:

$$P(R) = (0.25)(0.4) + (0.75)(0.1) = 0.10 + 0.075 = 0.175$$

$P(R) = 0.175$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Finds $P(S \cap R) = 0.1$ or $P(\overline{S} \cap R) = 0.075$, or equivalent merit	1

14 (d) (ii)

$$P(S \mid R) = \frac{P(S \cap R)}{P(R)} = \frac{0.10}{0.175} = \frac{4}{7}$$

$\frac{4}{7}$, about 0.571.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Writes $P(S \mid R) = \frac{P(S \cap R)}{P(R)}$ with a correct numerator or denominator, or equivalent merit	1

$P(S \mid R)$ is not $P(R \mid S) = 0.4$. The condition is the rough surf, so the denominator is $P(R) = 0.175$, not $P(S) = 0.25$.

What this paper examined

Focus area	Section I	Section II	Marks	Target
Functions (MAV-11-01 to 03)	3	15	**18**	18
Trigonometric functions (MAV-11-04, 05)	2	15	**17**	17
Calculus (MAV-11-06)	3	15	**18**	18
Exponential and logarithmic functions (MAV-11-07, 08)	1	8	**9**	9
Statistical analysis (MAV-11-09, 10)	1	7	**8**	8
Total	**10**	**60**	**70**	**70**

Where each focus area was examined:

- **Functions (MAV-11-01 to 03):** Questions 1, 2 and 3; 11 (a) algebraic fractions; 11 (b) surds; 11 (c) inverse variation; 13 (a) an absolute value equation and its graph; 13 (b) a circle.
- **Trigonometric functions (MAV-11-04, 05):** Questions 4 and 5; 11 (d) exact values of any angle; 11 (e) a trigonometric equation; 12 (d) bearings with the cosine and sine rules; 13 (c) a segment area in radians; 14 (c) an identity.
- **Calculus (MAV-11-06):** Questions 6, 7 and 8; 12 (a) first principles; 12 (b) chain and quotient rules; 12 (c) stationary points; 13 (d) velocity, rest and distance.

- **Exponential and logarithmic functions (MAV-11-07, 08):** Question 9; 14 (a) an exponential model; 14 (b) change of base and a logarithmic equation.
- **Statistical analysis (MAV-11-09, 10):** Question 10; 11 (f) a two-set count; 12 (e) a continuous random variable; 14 (d) a tree and conditional probability.

The questions rise in difficulty through the paper, and within each question the last parts are the hardest. Use the table to find the focus area where you lost the most marks, and work through that chapter of the Year 11 Complete Textbook before you sit Practice Paper 2.

End of worked solutions.

About this resource

MA99 EDUCATION

Scope

NSW Mathematics Advanced, Year 11. Written to the NSW Mathematics Advanced 11-12 Syllabus (2024), all Year 11 focus areas, for students sitting the HSC in 2028 and later years. Every value and solution was re-derived independently by computer algebra before printing. Contexts in the questions are invented unless a source is named.

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