MA99 EDUCATION

Advanced Mechanics: Topic Notes 05

NSW PHYSICS STAGE 6 SYLLABUS 2017 FOR HSC 2027

Module 5 takes the mechanics of Year 11 and applies it where the motion is two-dimensional. Nothing new is added to Newton's laws. What changes is that the force and the velocity are no longer along the same line, so you have to resolve, and the marks go to students who resolve before they substitute.

1. The two-minute diagnostic

Ten statements. Tick the ones that are true of you right now, honestly, then read the row your score lands in.

- ☐ I can state, without checking, what stays constant in projectile motion.
- ☐ I always work the vertical direction before the horizontal one.
- ☐ I can explain why a satellite at constant speed is still accelerating.
- ☐ I can derive $\tan\theta = v^2/rg$ for a banked track.
- ☐ I know the difference between mass, weight and gravitational field strength.
- ☐ I can get orbital speed from Newton's law of universal gravitation in two lines.
- ☐ I can state Kepler's third law in the form r cubed over T squared.
- ☐ I know why gravitational potential energy is negative.
- ☐ I can calculate the total energy of an orbit and say what its sign means.
- ☐ I write a direction on every vector answer without being told.

1.1 Interpreting your score

Count the ticks and read the matching row.

TICKS	WHAT IT MEANS	GO HERE FIRST
0 to 3	The content is not in place yet, and no amount of practice fixes that.	Sections 2 to 6, in order. Cover each worked answer before you read it.
4 to 6	You know the ideas and lose marks on the two-dimensional ones.	Sections 3 and 5, then the practice questions for those two.
7 to 9	Your gap is speed under pressure, not understanding.	The drill in section 7, weekly, then a full paper with a clock.
10	You are ready. Reading this document again will not add a mark.	Past papers under time. Come back only to check a formula.

An inflated score sends you to the wrong section, which costs more time than an accurate score would have.

Date this. Run the same statements again at the end of term and compare the result with this one to measure progress.

First run. Date: _____ **Score:** _____

End of term. Date: _____ **Score:** _____

2. Projectile motion

A projectile is any object moving only under gravity, once it has been launched. Air resistance is ignored at this level, which is stated in the question rather than assumed by you.

The whole of projectile motion rests on one idea. The horizontal and vertical motions are independent. Gravity acts vertically, so it changes the vertical velocity and leaves the horizontal velocity alone.

- Horizontally: acceleration is zero, so the velocity is constant and x = ux t.
- Vertically: acceleration is g = 9.8 m/s^2 downward, so the Year 11 equations of motion apply unchanged.
- The time of flight is shared. It is the only quantity that appears in both directions, which is what lets you move between them.

Key idea

Resolve the launch velocity first: ux = u cos θ and uy = u sin θ.

Work the vertical direction to find the time, then carry that time across. Choose a positive direction before you start and keep it for the whole question.

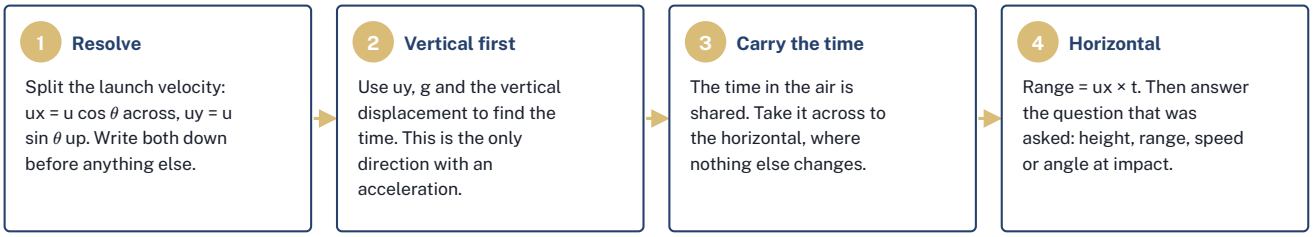


Figure 1 · The order to work a projectile question. The time in the air is the bridge between the vertical and the horizontal.

Worked example · 1

The Wyndarra launch rail fires a test mass at u = 25 m/s at 30 degrees above the horizontal, from ground level. Find the time of flight, the maximum height and the range.

Resolve: ux = 25 cos 30 = 21.7 m/s, uy = 25 sin 30 = 12.5 m/s

Time to the top: the vertical velocity falls to zero, so t = uy / g = 12.5 / 9.8 = 1.28 s

Time of flight: T = 2t = 2.55 s, because the flight is symmetric about the top

Maximum height: H = uy^2 / (2g) = 12.5^2 / 19.6 = 7.97 m

Range: R = ux × T = 21.65 × 2.551 = 55.2 m

ANSWER T = 2.55 s, H = 7.97 m, R = 55.2 m

Check it another way. The range of a projectile launched and landing at the same height is R = u^2 sin 2 θ / g. Here R = 25^2 × sin 60 / 9.8 = 55.2 m, which agrees. A second route to the same number is worth the thirty seconds it costs in an exam.

Worked example · 2

A package is released from the Dish 3 gantry, 45 m above the ground, from a trolley moving horizontally at 18 m/s. Find where it lands and how fast it is going on impact.

Vertical, taking down as positive: uy = 0, so 45 = ½ × 9.8 × t^2

t^2 = 90 / 9.8 = 9.18, so t = 3.03 s

Horizontal: x = 18 × 3.03 = 54.5 m from the release point

Impact vertical velocity: vy = gt = 9.8 × 3.03 = 29.7 m/s downward

Resultant speed: v = √(18^2 + 29.7^2) = √1206 = 34.7 m/s

Direction: θ = tan^{-1}(29.7 / 18) = 58.8 degrees below the horizontal

ANSWER it lands 54.5 m away at 34.7 m/s, 58.8 degrees below the horizontal

> The horizontal speed at impact is still 18 m/s. It was never going to be anything else, and every year somebody changes it.

The four things examiners look for

The launch velocity resolved into components before anything is substituted. A stated sign convention, used consistently.

The time found from the vertical motion, not guessed.

A final answer carrying both a magnitude and a direction.

3. Uniform circular motion

An object in uniform circular motion travels a circular path at constant speed. Its velocity is not constant, because the direction changes every instant, and a changing velocity is an acceleration. That acceleration points toward the centre of the circle. It is called the centripetal acceleration, and the net force producing it is the centripetal force. Centripetal is a description of a direction, not a new kind of force: the actual force is tension, gravity, friction or a normal force.

Key idea

$a = v^2 / r = \omega^2 r$, directed toward the centre.

$F = mv^2 / r = m \omega^2 r$, also toward the centre.

$v = 2 \pi r / T$, where T is the period of one full revolution.

The speed is constant, so the centripetal force does no work on the object.

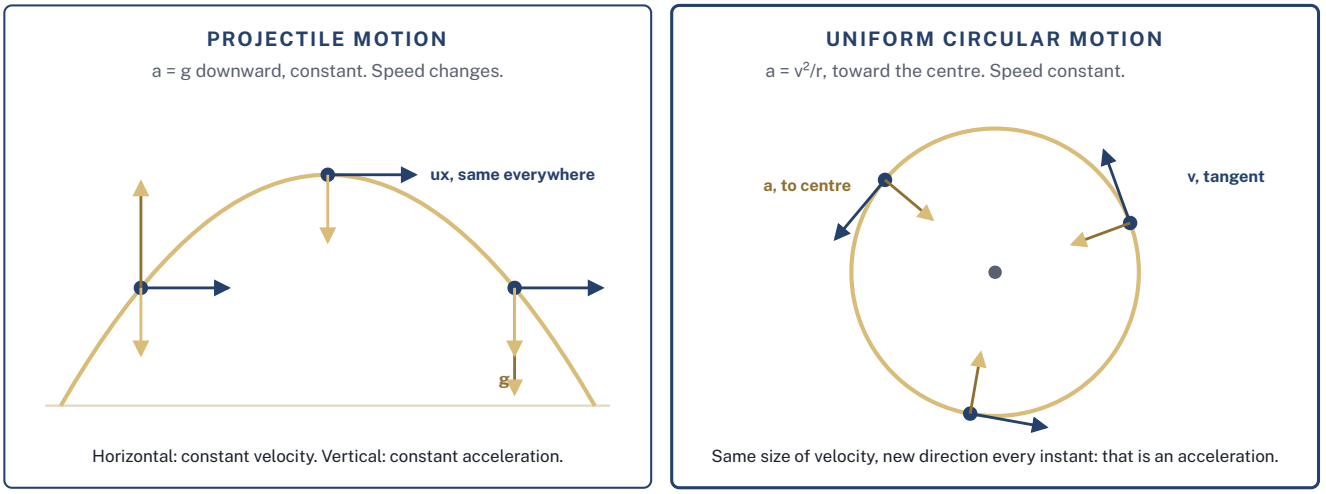


Figure 2 · Projectile motion against uniform circular motion. Both have a constant acceleration in size, and only one has an unchanging speed.

Worked example · 3

The 2.4 m centrifuge arm swings a 3.0 kg instrument package in a horizontal circle at 45 revolutions each minute. Find the angular velocity, the speed, the centripetal acceleration and the force on the package.

Period: T = 60 / 45 = 1.33 s

Angular velocity: $\omega = 2\pi / T = 2\pi \times 45 / 60 = 4.71$ rad/s

Speed: $v = \omega r = 4.71 \times 2.4 = 11.3$ m/s

Acceleration: $a = \omega^2 r = 4.71^2 \times 2.4 = 53.3$ m/s^2, toward the axis

Force: $F = ma = 3.0 \times 53.3 = 160$ N, toward the axis

ANSWER $\omega = 4.71$ rad/s, $v = 11.3$ m/s, $a = 53.3$ m/s^2, $F = 160$ N

Note the size of that. 53.3 m/s^2 is about 5.4 times g, on a 2.4 m arm at a speed a person could cycle. Centripetal acceleration grows with the square of the speed, which is why the numbers in this topic get large quickly.

A common and expensive error

There is no outward force on an object in circular motion. What passengers feel as an outward push is their own inertia carrying them straight on while the vehicle turns underneath them.

A free body diagram in this module has no arrow pointing away from the centre. Drawing one costs the mark for the diagram and usually the marks that follow from it.

4. Circular motion in practice

4.1 Banked tracks

On a flat road, the centripetal force for a turn comes from friction. On a banked track the road is tilted, so a component of the normal force points toward the centre of the turn and does the job instead. At the design speed no friction is needed at all.

RESOLVE
two axes, N in both

Vertical: $N \cos\theta = mg$

Horizontal: $N \sin\theta = mv^2 / r$

DIVIDE
N and m both cancel

Divide: $\tan\theta = v^2 / rg$

Rearrange: $v = \sqrt{(rg \tan\theta)}$

SUBSTITUTE
numbers last

Substitute: $v = \sqrt{(80 \times 9.8 \times \tan 18)}$

$v = 16.0$ m/s

No friction is needed at exactly this speed. Faster, and friction must act down the slope.

Figure 3 · Anatomy of a banked track solution. Resolving the normal force into two axes, then dividing the equations, removes both N and m at once.

Worked example · 4

The Wyndarra banked curve has radius 80 m and is banked at 18 degrees. Find the design speed, the speed at which no friction is required.

Vertical: $N \cos 18 = mg$

Horizontal: $N \sin 18 = mv^2 / r$

Divide the second equation by the first: $\tan 18 = v^2 / rg$

Rearrange: $v = \sqrt{(rg \tan 18)} = \sqrt{(80 \times 9.8 \times 0.325)}$

$v = \sqrt{254.7} = 16.0$ m/s

ANSWER the design speed is 16.0 m/s, about 57 km/h, and it does not depend on mass

> The mass cancels. That is why one banked curve suits a motorbike and a truck at the same speed, and it is worth a mark to say so.

4.2 Torque

Torque is the turning effect of a force about a pivot. It is $\tau = rF \sin \theta$, where r is the distance from the pivot to the point where the force acts and θ is the angle between that line and the force. Torque is measured in newton metres and it is a vector.

- The torque is largest when the force is applied perpendicular to the arm, because $\sin 90 = 1$.
- A force applied along the arm produces no torque at all, because $\sin 0 = 0$.

Worked example · 5

The drive motor on Dish 3 applies a force of 340 N perpendicular to a drive arm 0.85 m from the axis. Find the torque.

Formula: $\tau = rF \sin \theta$, with $\theta = 90$ degrees so $\sin \theta = 1$

Substitute: $\tau = 0.85 \times 340 \times 1$

ANSWER $\tau = 289$ N·m

5. Motion in gravitational fields

Newton’s law of universal gravitation says every mass attracts every other mass. The force is proportional to each mass and falls off with the square of the distance between their centres.

Newton's law of universal gravitation

$$F = GMm / r^2$$

$G = 6.67 \times 10^{-11}$ N·m^2/kg^2, the universal gravitational constant.

M and m are the two masses in kilograms, and r is the distance between their centres in metres, not the distance between their surfaces.

The force on each mass is the same size and opposite in direction, by Newton’s third law.

Dividing through by the small mass gives the gravitational field strength, $g = GM / r^2$. This is the force each kilogram feels, in N/kg, and it is the same number as the acceleration in free fall, in m/s^2.

Worked example · 6

Wyndarra 1 orbits 620 km above the Earth's surface. Take the radius of the Earth as 6.371×10^6 m and its mass as 6.0×10^{24} kg. Find the gravitational field strength there.

Orbital radius: $r = 6.371 \times 10^6 + 6.2 \times 10^5 = 6.99 \times 10^6$ m

Formula: $g = GM / r^2$

Substitute: $g = (6.67 \times 10^{-11} \times 6.0 \times 10^{24}) / (6.99 \times 10^6)^2$

$g = 4.00 \times 10^{14} / 4.89 \times 10^{13}$

ANSWER $g = 8.19$ m/s^2, about 84 per cent of its value at the surface

Why astronauts float, and why it is not zero gravity

The field strength in low Earth orbit is about 8.2 m/s^2, not far short of the 9.8 m/s^2 at the surface. Gravity is very much present.

Astronauts float because the station and everything in it are in free fall together, accelerating toward the Earth at the same rate. There is no normal force between an astronaut and the floor, and it is the normal force that a body registers as weight.

Say apparent weightlessness or free fall. Saying zero gravity in an extended response will cost you the mark.

5.1 Orbital speed and Kepler's third law

For a circular orbit, the gravitational force provides exactly the centripetal force. Setting one equal to the other and cancelling gives everything else in the module.

The derivation worth memorising

Start: $GMm / r^2 = mv^2 / r$

The orbiting mass m cancels from both sides, and one factor of r cancels.

Result: $v^2 = GM / r$, so $v = \sqrt{(GM / r)}$

Substituting $v = 2\pi r / T$ and rearranging gives $r^3 / T^2 = GM / 4\pi^2$, which is Kepler's third law. The orbiting mass is absent from both results. A heavy satellite and a light one at the same radius have the same speed and the same period.

Worked example · 7

Find the orbital speed and period of Wyndarra 1, at $r = 6.99 \times 10^6$ m.

Speed: $v = \sqrt{(GM / r)} = \sqrt{(4.00 \times 10^{14} / 6.99 \times 10^6)}$

$v = \sqrt{(5.72 \times 10^7)} = 7.57 \times 10^3$ m/s

Period: $T = 2\pi r / v = (2\pi \times 6.99 \times 10^6) / 7.57 \times 10^3$

$T = 5.81 \times 10^3$ s, which is 96.8 minutes

ANSWER $v = 7.57 \times 10^3$ m/s and $T = 5.81 \times 10^3$ s, about 97 minutes per orbit

Cross check with Kepler: $T = \sqrt{(4\pi^2 r^3 / GM)} = 5.81 \times 10^3$ s, which agrees

6. Energy in a gravitational field

Gravitational potential energy is defined as the work done to bring a mass from an infinite distance to a point in the field. Zero is placed at infinity, where the masses no longer interact, and since gravity does positive work on the way in, the potential energy everywhere else is negative.

The two energy expressions

U = −GMm / r, in joules, negative everywhere and approaching zero far away.

Note the single r in the denominator. The force has r squared, the energy has r. Mixing these is the single most common error in this module.

Near the surface only, the Year 11 form Δ U = mg Δ h is a good approximation, because g barely changes over a few hundred metres.

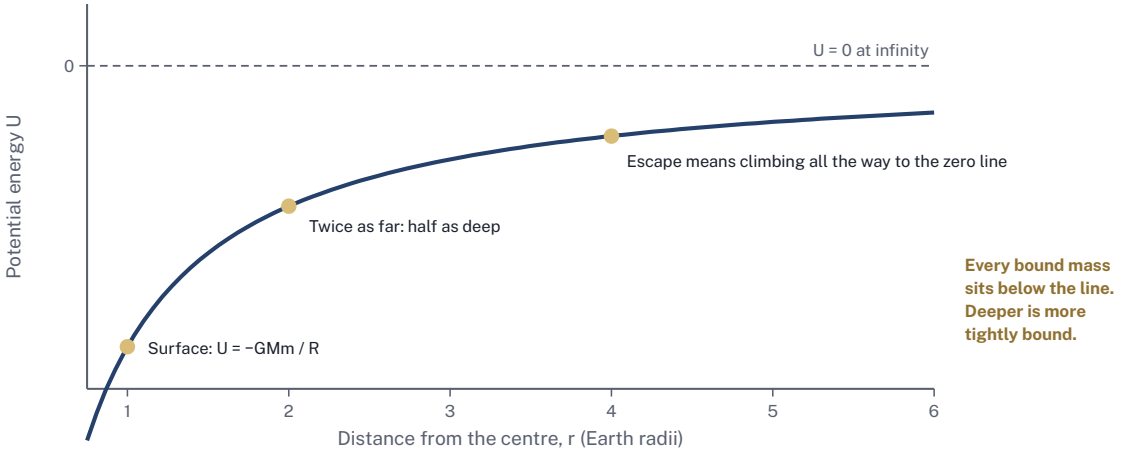


Figure 4 · The potential energy well. Every bound mass sits below the zero line, and the deeper it sits the more tightly it is bound.

Worked example · 8

Wyndarra 1 has mass 240 kg and orbits at r = 6.99 × 10^6 m. Find its kinetic energy, its potential energy and its total energy.

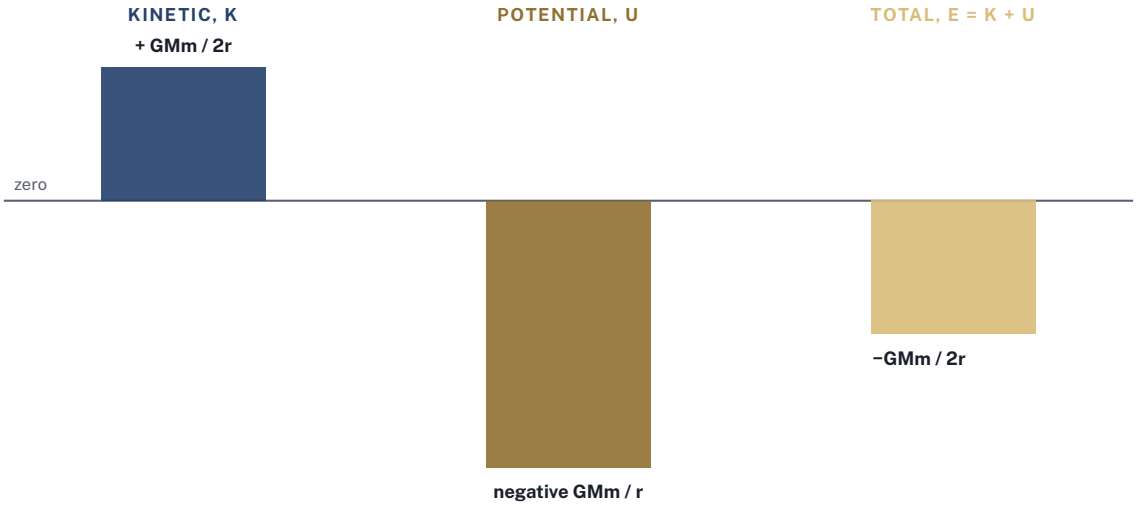
Kinetic: Ek = ½mv^2 = ½ × 240 × 5.72 × 10^7 = 6.87 × 10^9 J

Potential: U = negative GMm / r = negative (4.00 × 10^{14} × 240) / 6.99 × 10^6

U = negative 1.37 × 10^{10} J

Total: E = Ek + U = 6.87 × 10^9 + (negative 1.37 × 10^{10}) = negative 6.87 × 10^9 J

ANSWER the total energy is negative 6.87 × 10^9 J, equal in size to the kinetic energy



U is exactly twice the size of K and opposite in sign, so the total is negative, and equal in size to K.

Figure 5 · The energy of a circular orbit. The potential energy is exactly twice the size of the kinetic energy, and negative.

Three results that fall out of that

In any circular orbit, U = −2Ek, and the total energy E = −GMm / 2r.

A negative total energy means the mass is bound. Zero or positive means it escapes.

Moving to a higher orbit raises the total energy toward zero, but the satellite travels more slowly there. Higher orbit, more energy, less speed.

6.1 Escape velocity

Escape velocity is the launch speed at which a mass just reaches infinity with no speed left. Set the total energy to zero and solve.

Escape velocity

$$\tfrac{1}{2}mv^2 + (-GMm / r) = 0, \text{ so } v = \sqrt{(2GM / r)}$$

For the Earth's surface: $v = \sqrt{(2 \times 4.00 \times 10^{14} / 6.371 \times 10^{6})} = 1.12 \times 10^{4}$ m/s

That is about 11.2 km/s, and it does not depend on the mass being launched.

It is √2 times the orbital speed at the same radius, a relationship worth knowing.

It is a speed, not a direction. Any direction that misses the planet will do, which is why the word velocity in escape velocity is a historical accident rather than a description.

7. Practice under time

7.1 The twelve-minute drill

Twelve minutes, once a week, no notes and no calculator for the first two. The point is not to learn the content. It is to make retrieval fast enough that the exam does not take it away from you.

STEP	MINUTES	WHAT YOU DO
1	2	State what is constant and what changes in projectile motion, in both directions, from memory.
2	2	Write the four Module 5 formulae for circular motion without looking.
3	2	Derive v = √ (GM / r) from the two force expressions, in full.
4	2	Wyndarra 1 moves to a 900 km orbit. Recalculate v and T.
5	2	Explain to an empty room why a satellite at constant speed is accelerating. Out loud, in under thirty seconds.
6	2	State the sign of the total energy of a bound orbit and say why.

> If step 3 takes longer than ninety seconds, that is your revision for the week, and nothing else on this list is.

7.2 Fill this in from memory, then check it

A blank grid is intimidating and a prompted one gets finished. The left column is done for you.

QUANTITY	SYMBOL AND UNIT	WHAT CHANGES IT
Centripetal acceleration		
Centripetal force		

QUANTITY	SYMBOL AND UNIT	WHAT CHANGES IT
Period		
Angular velocity		
Torque		
Field strength		
Orbital speed		
Total orbital energy		

Your module 5 progress, dated

Come back every few weeks and date it.

Improvement is invisible from the inside, which is why students who are getting better so often believe they are not.

DATE	WHAT I SAT	MARK AND THE ONE THING TO FIX

Fill one row every time you sit a set. A tracker with three rows in it is worth more than a perfect one you never started, because the pattern you are looking for only shows up across attempts, not inside one.

About this resource

MA99 EDUCATION

Scope

NSW Physics, Year 12. Written to the NSW Physics Stage 6 Syllabus (2017), Module 5, for students sitting the HSC in 2027. A new Physics syllabus is first examined in HSC 2028. Many worked contexts use the Wyndarra site and its instruments, an invented facility, and every number was recomputed from its inputs before printing.

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MA99 EDUCATION

Advanced Mechanics: Practice Questions

05

NSW PHYSICS STAGE 6 SYLLABUS 2017 FOR HSC 2027

1. Instructions

How to use this

Work it as a test, not as a reading exercise. Forty minutes, calculator, NESA data sheet, no notes and no solutions in front of you. Mark it yourself against the guide at the back before you read any solution. Marking shows what a marker sees. Use $g = 9.8\ m/s^2$, $G = 6.67 \times 10^{-11}\ N{\cdot}m^2/kg^2$, the mass of the Earth as 6.0×10^{24} kg and its radius as 6.371×10^6 m.

BAND	QUESTIONS	WHAT THEY TEST
Band 4 to 5	Questions 1 to 9	Recall and one step substitution.
Band 5 to 6	Questions 10 to 13	Two steps, or a reason attached.
Band 6	Questions 14 and 15	Derivation, judgement and communication.

2. Section A · Multiple choice

Eight marks. One mark each. Circle one letter.

Question 1

1 MARK

Which quantity stays constant throughout the flight of a projectile, with air resistance neglected?

A the vertical velocity
B the horizontal velocity
C the speed
D the momentum

Question 2

1 MARK

The orbital speed of a satellite in a circular orbit does not depend on

A the mass of the satellite
B the mass of the central body
C the orbital radius
D the gravitational constant

Question 3

1 MARK

An object moves in a circle of radius 36 m at 12 m/s. Its centripetal acceleration is

A $0.33\ m/s^2$
B $4.0\ m/s^2$
C $144\ m/s^2$
D $432\ m/s^2$

Question 4

1 MARK

For a vehicle on a banked track at the design speed, the centripetal force is supplied by

A friction alone
B the horizontal component of the normal force
C the vertical component of the normal force
D the weight of the vehicle

Question 5

1 MARK

Gravitational potential energy is negative because

A gravity is a repulsive force

B the zero of potential energy is defined at an infinite separation

C the satellite is below the surface of the Earth

D kinetic energy is always larger

Question 6

1 MARK

If the radius of a circular orbit is doubled, the period is multiplied by about

A 1.41

B 2.00

C 2.83

D 4.00

Question 7

1 MARK

A force of 20 N acts 0.50 m from a pivot, at 30 degrees to the arm. The torque is

A 5.0 N·m

B 8.7 N·m

C 10 N·m

D 20 N·m

Question 8

1 MARK

Escape velocity from a planet does not depend on

A the mass of the escaping object

B the mass of the planet

C the radius of the planet

D the gravitational constant

3. Section B · Short answer and calculation

Twenty six marks. Show all working. A correct answer with no working earns fewer marks than the working alone.

Question 9

4 MARKS

The Wyndarra launch rail fires a test mass at 25 m/s at 60 degrees above the horizontal from level ground. Calculate the maximum height reached and the horizontal range.

Question 10

5 MARKS

The 2.4 m centrifuge arm must subject a 3.0 kg instrument package to a centripetal acceleration of 8.0 times g. Calculate the angular velocity required, the corresponding rate in revolutions each minute, and the force on the package.

Question 11

4 MARKS

The Wyndarra banked curve has radius 80 m and is banked at 18 degrees. A 1200 kg vehicle takes the curve at 22 m/s. Calculate the centripetal force required, and explain what supplies the part of it that the banking does not.

Question 12

6 MARKS

Wyndarra 1 has a mass of 240 kg and orbits 620 km above the Earth's surface. It is moved to a circular orbit 1200 km above the surface. Calculate the new orbital speed and period, and state with a reason whether energy had to be supplied.

Question 13

3 MARKS

A drive beam on Dish 3 is pivoted at one point. A force of 450 N acts perpendicular to the beam 0.80 m from the pivot on one side. Calculate the perpendicular force needed 2.40 m from the pivot on the other side to hold the beam in rotational equilibrium.

Question 14

4 MARKS

A communications satellite is to be placed in a geostationary orbit. Using Newton's law of universal gravitation and the requirements of circular motion, derive an expression for the orbital radius in terms of the period, and use it to calculate the radius and the altitude of a geostationary orbit. Take the period as 8.6164×10^4 s.

4. Section C · Extended response

Question 15

6 MARKS

A satellite is transferred from a low circular orbit to a higher circular orbit. Explain why it travels more slowly in the higher orbit and yet energy must be supplied to put it there. Support your explanation with the relevant relationships.

> Forty minutes. Stop when the forty minutes are up.

5. Worked solutions

Mark your own paper against these before you read them as explanations. The marks in brackets show where each one is earned.

Section A

Q	ANSWER	WHY THE OTHERS FAIL
1	B, the horizontal velocity	Gravity acts vertically only, so it changes the vertical velocity and the speed, and leaves the horizontal velocity untouched.
2	A, the mass of the satellite	In $GMm / r^2 = mv^2 / r$ the satellite mass cancels, leaving $v = \sqrt{(GM / r)}$.
3	B, 4.0 m/s^2	$a = v^2 / r = 144 / 36 = 4.0$. Option C is v^2 alone and option D is $v \times r$.
4	B, the horizontal component of the normal force	At the design speed no friction is needed. The vertical component balances the weight instead.
5	B, the zero is defined at an infinite separation	Gravity does positive work as a mass falls inward from infinity, so the potential energy becomes negative.
6	C, 2.83	T^2 is proportional to r^3, so T is proportional to $r^{1.5}$, and $2^{1.5} = 2.83$.
7	A, 5.0 N·m	$\tau = rF \sin\theta = 0.50 \times 20 \times \sin 30 = 5.0$. Option C forgets the sine.
8	A, the mass of the escaping object	$v = \sqrt{(2GM / r)}$ contains the planet's mass, not the escaping mass.

Section B

QUESTION 9 · 4 MARKS

Resolve: $uy = 25 \sin 60 = 21.7$ m/s, $ux = 25 \cos 60 = 12.5$ m/s. [1]

Maximum height: $H = uy^2 / 2g = 21.7^2 / 19.6 = 23.9$ m. [1]

Time of flight: $T = 2uy / g = 2 \times 21.7 / 9.8 = 4.42$ s. [1]

Range: $R = ux \times T = 12.5 \times 4.42 = 55.2$ m. [1]

Answer: H = 23.9 m and R = 55.2 m.

QUESTION 10 · 5 MARKS

Required acceleration: $a = 8.0 \times 9.8 = 78.4$ m/s^2. [1]

From $a = \omega^2 r$: $\omega = \sqrt{(a / r)} = \sqrt{(78.4 / 2.4)} = \sqrt{32.7} = 5.72$ rad/s. [1]

Rate: $5.72 \times 60 / 2\pi = 54.6$ revolutions each minute. [1]

Speed, if wanted: $v = \omega r = 5.72 \times 2.4 = 13.7$ m/s.

Force: $F = ma = 3.0 \times 78.4 = 235$ N, directed toward the axis. [2]

Answer: $\omega = 5.72$ rad/s, about 54.6 rpm, and F = 235 N toward the axis.

QUESTION 11 · 4 MARKS

Required centripetal force: $F = mv^2 / r = 1200 \times 22^2 / 80$. [1]

$F = 1200 \times 484 / 80 = 7260$ N, directed horizontally toward the centre of the curve. [1]

The design speed of this curve is $\sqrt{(rg \tan 18)} = 16.0$ m/s. [1]

At 22 m/s the vehicle is above the design speed, so the horizontal component of the normal force is no longer enough on its own. Sideways friction between the tyres and the road supplies the rest, acting down the slope toward the centre of the curve. If the required friction exceeds what the tyres can provide, the vehicle slides outward. [1]

QUESTION 12 · 6 MARKS

New radius: $r = 6.371 \times 10^6 + 1.2 \times 10^6 = 7.57 \times 10^6$ m. [1]

Speed: $v = \sqrt{(GM / r)} = \sqrt{(4.00 \times 10^{14} / 7.57 \times 10^{6})} = 7.27 \times 10^{3}$ m/s. [1]

Period: $T = 2\pi r / v = 2\pi \times 7.57 \times 10^{6} / 7.27 \times 10^{3} = 6.54 \times 10^{3}$ s, about 109 minutes. [1]

Energy before: $E = -GMm / 2r = -6.87 \times 10^{9}$ J. [1]

Energy after: $E = -6.34 \times 10^{9}$ J. [1]

The total energy has risen by 5.26×10^{8} J, so energy had to be supplied. The satellite is slower but less tightly bound: the potential energy rose by twice as much as the kinetic energy fell. [1]

QUESTION 13 · 3 MARKS

For rotational equilibrium the clockwise torque equals the anticlockwise torque. [1]

$450 \times 0.80 = F \times 2.40$ [1]

$F = 360 / 2.40 = 150$ N. [1]

QUESTION 14 · 4 MARKS

Gravitation supplies the centripetal force: $GMm / r^2 = mv^2 / r$, so $v^2 = GM / r$. [1]

For a circular orbit $v = 2\pi r / T$, so $4\pi^2 r^2 / T^2 = GM / r$. [1]

Rearranged: $r^3 = GMT^2 / 4\pi^2$, so $r = (GMT^2 / 4\pi^2)^{1/3}$. [1]

Substitute: $r = (4.00 \times 10^{14} \times (8.6164 \times 10^4)^2 / 39.48)^{1/3} = 4.22 \times 10^7$ m.

Altitude = 4.22×10^7 minus $6.371 \times 10^6 = 3.58 \times 10^7$ m, about 35 800 km. [1]

Section C

QUESTION 15 · 6 MARKS

A band 6 response makes four moves.

One. State the speed relationship. Equating gravitation to the centripetal requirement gives $GMm / r^2 = mv^2 / r$, so $v = \sqrt{(GM / r)}$. Speed therefore falls as the orbital radius rises, which is why the higher orbit is the slower one. (2) Two. State the energy relationship. $E_k = GMm / 2r$ and $U = -GMm / r$, so the total energy $E = -GMm / 2r$. (1) Three. Resolve the apparent contradiction. As r increases, $E = -GMm / 2r$ becomes less negative, which means the total energy has increased even though the speed has fallen. The kinetic energy drops by GMm/2 times the change in one over r, while the potential energy rises by twice that amount. The gain in potential energy outweighs the loss in kinetic energy, and the difference is the energy that must be supplied. (2) Four. Close on the physical meaning. A higher orbit is a less tightly bound orbit. Work must be done against gravity to get there, and the satellite arrives with more total energy and less speed. [1]

Marking guide

Marking guide

Section A: one mark each, no part marks. 8 marks. Section B: marks are shown against each line of the solutions. A correct final answer with no working attracts the answer mark only. 26 marks. Section C: 5 to 6 marks for both relationships stated correctly, the contradiction resolved quantitatively and a clear closing judgement. 3 to 4 marks for both relationships with a partial resolution. 1 to 2 marks for one relationship or a purely qualitative answer. 6 marks. Total: 40 marks.

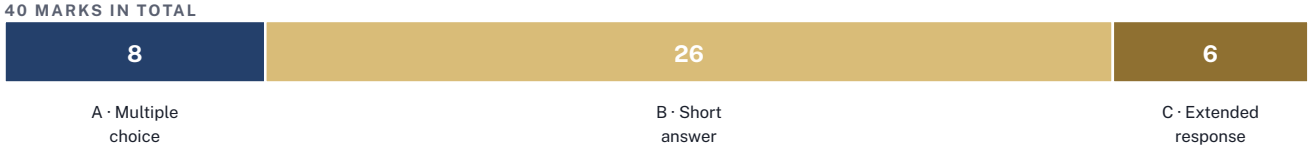


Figure 1 · How the 40 marks are split across the sections. Your time should follow the same split.

> Mark it before you read the solutions. Reading a solution feels like learning and is not.

Attempt record

Come back every few weeks and date it. Improvement is invisible from the inside, which is why students who are getting better so often believe they are not.

DATE	MARK OUT OF 40	THE ONE THING TO FIX NEXT TIME

Fill one row every time you sit a set. A tracker with three rows in it is worth more than a perfect one you never started, because the pattern you are looking for only shows up across attempts, not inside one.

About this resource

MA99 EDUCATION

Scope

NSW Physics, Year 12. Written to the NSW Physics Stage 6 Syllabus (2017), Module 5, for students sitting the HSC in 2027. A new Physics syllabus is first examined in HSC 2028. Many worked contexts use the Wyndarra site and its instruments, an invented facility, and every number was recomputed from its inputs before printing.

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