MA99 EDUCATION

Mathematics Extension 1: Practice Paper 1

01

NSW MATHEMATICS EXTENSION 1 11-12 SYLLABUS 2024

FOR HSC 2027 AND LATER

General Instructions

- Reading time: 10 minutes
- Working time: 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- The NESA Mathematics Extension 1 reference sheet may be used.
- From Question 11 on, show your mathematical reasoning and/or calculations in every answer

Total marks: 70

Section I · 10 marks

Questions 1 to 10

Suggested time: about 15 minutes

Section II · 60 marks

Questions 11 to 14

Suggested time: about 1 hour 30 minutes (a minute and a half per mark), which leaves about 15 minutes to check your work

Year 11 Mathematics Advanced and Extension 1 work is assumed, and questions can draw on it.

This paper examines the whole Year 12 Mathematics Extension 1 course of the 2024 syllabus, with about a fifth of the marks on the Year 11 course, as the HSC does. It is set at the level of a typical HSC paper. Questions with a setting are drawn from sport and recreation.

Marks may be deducted for careless or badly arranged work.

Section I · Answer Sheet

Shade ONE circle for each of Questions 1 to 10. If you change your mind, draw a cross through the incorrect answer and shade the new one.

1	A	B	C	D	2	A	B	C	D
3	A	B	C	D	4	A	B	C	D
5	A	B	C	D	6	A	B	C	D
7	A	B	C	D	8	A	B	C	D
9	A	B	C	D	10	A	B	C	D

Section I · 10 marks

Answer Questions 1 to 10 on the multiple choice answer sheet, spending about 15 minutes on this section.

Question 1

1 MARK

The points $P(1, 0, -2)$ and $Q(3, -2, -1)$ are given. What is the value of $\left|\overrightarrow{PQ}\right|$?

A $\sqrt{17}$

B 9

C 3

D $\sqrt{29}$

Question 2

1 MARK

Which of the following is equal to $\int \cos^2 2x \, dx$?

A $\dfrac{x}{2} + \dfrac{\sin 4x}{8} + c$

B $\dfrac{x}{2} - \dfrac{\sin 4x}{8} + c$

C $\dfrac{x}{2} + \dfrac{\sin 4x}{4} + c$

D $\dfrac{\cos^3 2x}{6} + c$

Question 3

1 MARK

A basketball coach chooses a starting five from a squad of 9 players. Two particular players in the squad cannot both be chosen. In how many ways can the starting five be chosen?

A 35

B 70

C 91

D 126

Question 4

1 MARK

What are the domain and range of $y = 4\sin^{-1}(3x)$?

A Domain $-3 \leq x \leq 3$range $-2\pi \leq y \leq 2\pi$

B Domain $-\frac{1}{3} \leq x \leq \frac{1}{3}$, range $-2\pi \leq y \leq 2\pi$

C Domain $-\frac{1}{3} \leq x \leq \frac{1}{3}$, range $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

D Domain $-\frac{1}{3} \leq x \leq \frac{1}{3}$, range $0 \leq y \leq 4\pi$

Question 5

1 MARK

A netball goal shooter scores from each attempt with probability 0.8, independently of her other attempts. She has 15 attempts in a quarter. Let X be the number of goals she scores. What is $\mathrm{Var}(X)$?

A 0.16

B 1.55

C 12

D 2.4

Question 6

1 MARK

Which of the following is the general solution of the differential equation $\frac{dy}{dx} = 3y - 12$, where A is a constant?

A $y = -4 + Ae^{3x}$

B $y = 4 + Ae^{3x}$

C $y = 12 + Ae^{3x}$

D $y = 4 + Ae^{-3x}$

Question 7

1 MARK

The vectors $\mathbf{a} = k\mathbf{i} + 2\mathbf{j}$ and $\mathbf{b} = 3\mathbf{i} + (k-5)\mathbf{j}$ are perpendicular. What is the value of k?

A $k = 2$

B $k = -2$

C $k = -1$ or $k = 6$

D $k = 1$

Question 8

1 MARK

A curve is defined by the parametric equations $x = t^3 + 1$ and $y = t^2 + 3t$. What is the value of $\frac{dy}{dx}$ at the point where $t = 1$?

A $\frac{3}{5}$

B 5

C 15

D $\frac{5}{3}$

Question 9

1 MARK

A student is proving by mathematical induction that $11^n - 4^n$ is divisible by 7 for all integers $n \geq 1$. In the inductive step she assumes that $11^k - 4^k = 7M$ for some integer M. Which expression is equal to $11^{k+1} - 4^{k+1}$?

A $77M$

B $77M - 15 \times 4^k$

C $77M + 7 \times 4^k$

D $77M + 15 \times 4^k$

Question 10

1 MARK

For which values of x is $\cos^{-1}(\cos x) = 2\pi - x$?

A $0 \leq x \leq \pi$

B $\pi \leq x \leq 2\pi$

C $-\pi \leq x \leq 0$

D $2\pi \leq x \leq 3\pi$

Section II · 60 marks

Answer Questions 11 to 14 in the spaces provided, spending about 1 hour 30 minutes on this section. Show your mathematical reasoning and/or calculations in every answer.

Question 11 · 15 marks

(a) Let $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 4\mathbf{j} - \mathbf{k}$. Find the angle between $\mathbf{a}$ and $\mathbf{b}$, correct to the nearest degree. 2 MARKS

(b) Use the substitution $u = 2x + 1$ to evaluate $\displaystyle\int_0^4 \frac{x}{\sqrt{2x+1}}\,dx$. 3 MARKS

(c) A basketballer makes each free throw with probability 0.75, independently of her other throws. At training she takes 8 free throws.

(i) Find the probability that she makes exactly 6 of the 8 free throws. Give your answer correct to three decimal places. 1 MARK

(ii) Find the probability that she makes at least 7 of the 8 free throws. Give your answer correct to three decimal places. 2 MARKS

(d) Find the exact value of $\tan\left(2\cos^{-1}\left(-\frac{3}{4}\right)\right)$. 2 MARKS

(e) The equation $x^3 - 3x^2 - 6x + 8 = 0$ has roots α, β and γ. Find the value of $\alpha^2 + \beta^2 + \gamma^2$. 2 MARKS

(f) Find the area of the region enclosed by the curves $y = 4 - x^2$ and $y = x^2 - 2x$. 3 MARKS

Question 12 · 15 marks

(a) Prove by mathematical induction that, for all integers $n \geq 1$, 3 MARKS

$$1 \times 2 + 2 \times 5 + 3 \times 8 + \cdots + n(3n-1) = n^2(n+1).$$

(b) In a soccer drill, a ball is passed along the flat ground of a training pitch. Its position vector t seconds after it is passed is $\mathbf{r}_B(t) = (2 + 6t)\mathbf{i} + (1 + 8t)\mathbf{j}$. A teammate starts running at the same instant, and her position vector is $\mathbf{r}_P(t) = (14 + 2t)\mathbf{i} + (-2 + 9t)\mathbf{j}$. Distances are in metres.

(i) Find the speed of the ball. 1 MARK

(ii) Show that the teammate reaches the ball, and find the time and the position at which this happens. 2 MARKS

π

(c) By first writing $\sqrt{3}\sin x + \cos x$ in the form $R\sin(x+\alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$, solve $\sqrt{3}\sin x + \cos x = 1$ for $0 \leq x \leq 2\pi$. 3 MARKS

(d) Find the exact value of $\displaystyle\int_0^2 \frac{dx}{12+3x^2}$. 2 MARKS

(e) Let $f(x) = x^5 + x^3 + 1$. The function f has an inverse function, and the point $(1, 3)$ lies on the graph of $y = f(x)$. Find the gradient of the tangent to the graph of $y = f^{-1}(x)$ at the point $(3, 1)$. 2 MARKS

(f) A hockey squad of 6 players is to be chosen from 7 forwards and 5 defenders. In how many ways can the squad be chosen if it must include at least 2 defenders? 2 MARKS

Question 13 · 15 marks

(a) Solve $\dfrac{x+5}{x-2} \leq 3$. 3 MARKS

(b) Find the coefficient of x^5 in the expansion of $\left(2x - \dfrac{3}{x}\right)^7$. 2 MARKS

(c) Sketch the graph of $y = \pi - 2\cos^{-1}\left(\dfrac{x}{2}\right)$, showing its domain and range, the coordinates of its endpoints and any intercepts with the axes. 3 MARKS

(d) A bottle of sports drink is taken from a fridge at 4°C and left on a bench at a sports ground, where the air temperature is a constant 24°C. The temperature T°C of the drink t minutes later satisfies $\dfrac{dT}{dt} = -k(T - 24)$, where k is a positive constant.

(i) Show that $T = 24 - 20e^{-kt}$ satisfies the differential equation and the initial condition. 1 MARK

(ii) After 10 minutes the drink is at 9°C. A player will not drink it once it is warmer than 18°C. Find, to the nearest minute, how long after leaving the fridge the drink reaches 18°C. 3 MARKS

(e) The region bounded by the curve $y = 2\cos x$ and the line $y = 1$, for $-\dfrac{\pi}{3} \leq x \leq \dfrac{\pi}{3}$, is rotated about the x-axis. Find the exact volume of the solid formed. 3 MARKS

Question 14 · 15 marks

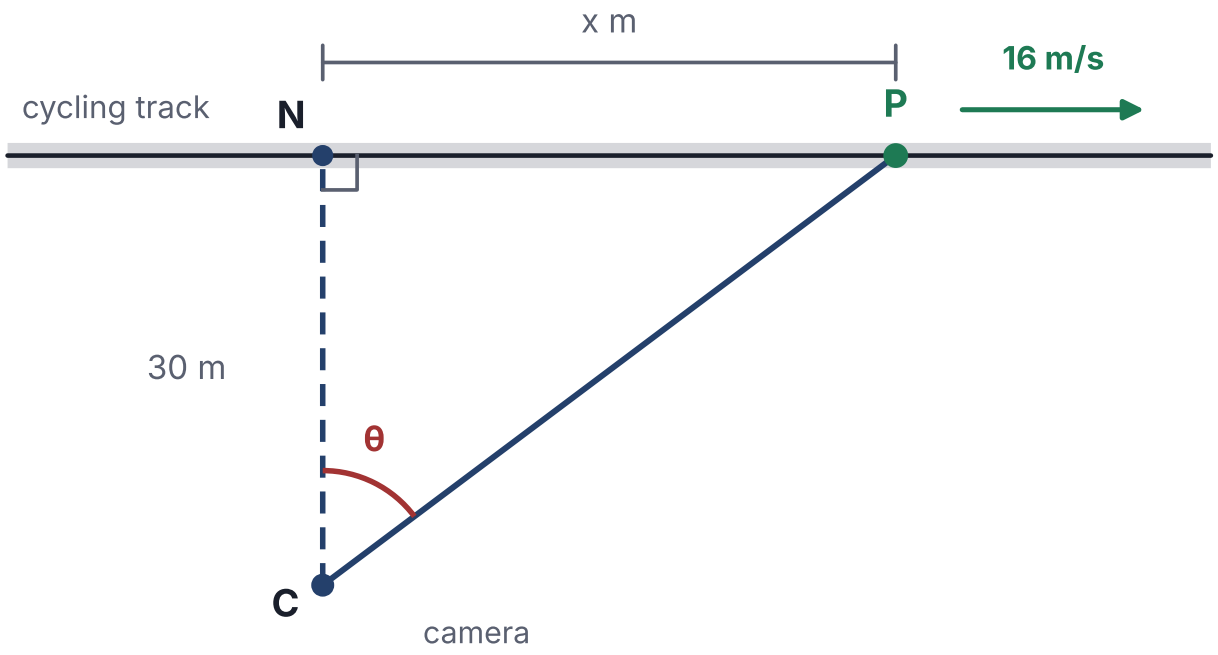
(a) Let $f(x) = 2(x-1)^2 + 3$ for $x \geq 1$ Find $f^{-1}(x)$ and state its domain. 2 MARKS

(b) The finishing times in a large 5 km fun run have mean 31 minutes and standard deviation 7.2 minutes. The distribution of the times is skewed to the right. A random sample of 36 runners is chosen, and $\overline{X}$ is the mean finishing time of the sample.

(i) Explain why $\overline{X}$ is approximately normally distributed, and state its mean and its standard deviation. 1 MARK

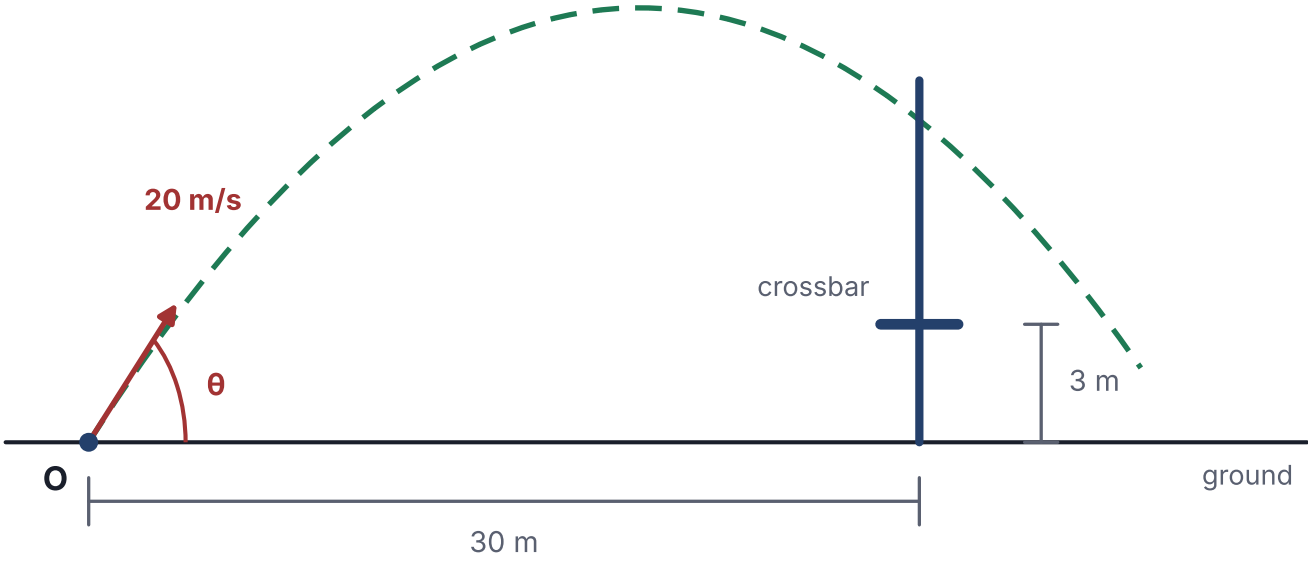
(ii) Find the probability that the sample mean is less than 29.2 minutes. You may use $P(Z \leq 1.5) = 0.9332$, where Z is a standard normal variable. 2 MARKS

(c) A television camera at C is 30 m from a straight cycling track, and N is the point on the track closest to C. A cyclist P rides along the track away from N at a constant 16 m/s, and the camera turns to follow her. Let $NP = x$ metres and let $\theta = \angle NCP$ radians, as shown. Find the rate at which θ is increasing at the instant the cyclist is 40 m past N. 3 MARKS



Question 14 (c) · the camera C and the cyclist P (not to scale)

(d) In rugby, a conversion kick is taken from the point O on level ground, 30 m in front of the goal posts and directly in line with them. The ball is kicked from O towards the posts with speed 20 m/s at an angle θ to the horizontal, where $0 < \theta < \dfrac{\pi}{2}$. To score, the ball must pass above the crossbar, which is 3 m above the ground. Model the ball as a point and ignore air resistance. Take O as the origin, with $\mathbf{i}$ and $\mathbf{j}$ unit vectors horizontally towards the posts and vertically upwards, and take the acceleration of the ball to be $\mathbf{a} = -10\mathbf{j}$.



Question 14 (d) · the conversion kick (not to scale)

(i) Show that the position vector of the ball t seconds after it is kicked is 2 MARKS

$$\mathbf{r}(t) = (20t\cos\theta)\mathbf{i} + \left(20t\sin\theta - 5t^{\,2}\right)\mathbf{j}.$$

(ii) Show that $y = x\tan\theta - \dfrac{x^2\left(1+\tan^2\theta\right)}{80}$ is the Cartesian equation of the path of the ball. 1 MARK

(iii) Find the values of θ for which the ball passes above the crossbar. Give the boundary angles in degrees, correct to one decimal place. 4 MARKS

End of paper.

About this resource

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Scope

NSW Mathematics Extension 1, Year 12. Written to the NSW Mathematics Extension 1 11-12 Syllabus (2024), an HSC style paper on the whole course: the Year 12 focus areas, with the Year 11 course as assumed knowledge, for students sitting the HSC in 2027 and later years. Every value and solution was re-derived independently by computer algebra before printing. Contexts in the questions are invented unless a source is named.

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MA99 EDUCATION

Mathematics Extension 1: Practice Paper 1 · Worked Solutions

01

NSW MATHEMATICS EXTENSION 1 11-12 SYLLABUS 2024 FOR HSC 2027 AND LATER

Worked solutions · Section I

1

$\overrightarrow{PQ} = \begin{pmatrix} 3-1 \\ -2-0 \\ -1-(-2) \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$, so $\left|\overrightarrow{PQ}\right| = \sqrt{4+4+1} = 3$.

(C)

(A) $\sqrt{17}$ comes from a sign slip in the third component, $-1-2=-3$ instead of $-1-(-2)=1$. (B) 9 is $\left|\overrightarrow{PQ}\right|^2$: the square root was not taken. (D) $\sqrt{29}$ adds the position vectors instead of subtracting them: $|(4,-2,-3)| = \sqrt{29}$.

2

With $n=2$ in $\cos^2 nx = \frac{1}{2}(1+\cos 2nx)$ (reference sheet):

$$\int \cos^2 2x\,dx = \int \frac{1}{2}(1+\cos 4x)\,dx = \frac{x}{2} + \frac{\sin 4x}{8} + c$$

(A)

(B) uses the identity for $\sin^2 2x$, with the minus sign. (C) drops the factor $\frac{1}{2}$ in front of $\cos 4x$, integrating $\frac{1}{2}+\cos 4x$. (D) treats $\cos^2 2x$ like a power of x, which ignores the chain rule; differentiating (D) does not give $\cos^2 2x$.

3

Count all teams and subtract those with both players. With both chosen, the other 3 come from the remaining 7:

$$^9C_5 - {}^7C_3 = 126 - 35 = 91$$

(C)

(A) 35 counts the teams that DO contain both players. (B) $70 = 2 \times {}^7C_4$ counts only the teams with exactly one of the two, forgetting the ${}^7C_5 = 21$ teams with neither. (D) 126 ignores the condition.

4

$\sin^{-1}(3x)$ needs $-1 \le 3x \le 1$so $-\frac{1}{3} \le x \le \frac{1}{3}$. Its values lie in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, and multiplying by 4 gives $-2\pi \le y \le 2\pi$.

(B)

(A) dilates the domain the wrong way: $3x$ compresses horizontally by a factor of 3. (C) forgets the vertical dilation by 4. (D) uses the range of $\cos^{-1}$, $[0,\pi]$, multiplied by 4.

5

$X \sim \text{Bin}(15, 0.8)$, so $\text{Var}(X) = np(1-p) = 15 \times 0.8 \times 0.2 = 2.4$.

(D)

(A) $0.16 = p(1-p)$ is the variance of one Bernoulli trial, not of 15. (B) $1.55 \approx \sqrt{2.4}$ is the standard deviation. (C) $12 = np$ is the mean.

6

$\frac{dy}{dx} = 3y - 12 = 3(y - 4)$ has the form $\frac{dQ}{dt} = k(Q - P)$ with $k = 3$ and $P = 4$, so $y = 4 + Ae^{3x}$. Check: $\frac{dy}{dx} = 3Ae^{3x} = 3(y - 4)$.

(B)

(A) takes the sign of P from the -4 inside $y - 4$. (C) reads $P = 12$ from $3y - 12$ without first taking out the factor 3. (D) changes the sign of k: $y = 4 + Ae^{-3x}$ satisfies $\frac{dy}{dx} = -3(y - 4)$.

7

$$\mathbf{a} \cdot \mathbf{b} = 3k + 2(k - 5) = 5k - 10$$

$$5k - 10 = 0 \implies k = 2 \quad \text{perpendicular: } \mathbf{a} \cdot \mathbf{b} = 0$$

(A)

(B) is a sign slip, $3k + 2k + 10 = 0$. (C) uses the condition for PARALLEL vectors, $k(k - 5) = 2 \times 3$, which gives $k = -1$ or $k = 6$. (D) expands $2(k - 5)$ as $2k - 5$.

8

$$\frac{dx}{dt} = 3t^2, \qquad \frac{dy}{dt} = 2t + 3$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t + 3}{3t^2} = \frac{2 + 3}{3} = \frac{5}{3} \quad \text{at } t = 1$$

(D)

(A) divides the wrong way, $\frac{dx/dt}{dy/dt}$. (B) 5 is $\frac{dy}{dt}$ alone. (C) 15 multiplies $\frac{dy}{dt}$ by $\frac{dx}{dt}$ instead of dividing.

9

From the assumption, $11^k = 7M + 4^k$. Then

$$11^{k+1} - 4^{k+1} = 11\left(7M + 4^k\right) - 4 \times 4^k = 77M + 11 \times 4^k - 4 \times 4^k = 77M + 7 \times 4^k$$

which is $7\left(11M + 4^k\right)$, a multiple of 7. (C)

(A) writes $11^{k+1} - 4^{k+1} = 11\left(11^k - 4^k\right)$, but 11×4^k is not 4^{k+1}. (B) rearranges the assumption with the wrong sign, $11^k = 7M - 4^k$. (D) adds 4^{k+1} instead of subtracting it.

10

$\cos^{-1}(\cos x)$ always lies in $[0, \pi]$. For $\pi \le x \le 2\pi$ the angle $2\pi - x$ lies in $[0, \pi]$ and $\cos(2\pi - x) = \cos x$, so $\cos^{-1}(\cos x) = 2\pi - x$ there.

(B)

(A) is where $\cos^{-1}(\cos x) = x$. (C) is where $\cos^{-1}(\cos x) = -x$ (there $-x$ lies in $[0, \pi]$). (D) is where it equals $x - 2\pi$; at $x = 3\pi$, for example, $2\pi - x = -\pi$ is negative, which $\cos^{-1}$ never is.

Section I key

1 C · 2 A · 3 C · 4 B · 5 D · 6 B · 7 A · 8 D · 9 C · 10 B

For each item you missed, find the distractor you chose above: it names the exact error that produces it. Write that error down before you look at Section II.

Worked solutions · Section II

Question 11

11 (a)

$$\mathbf{a}\cdot\mathbf{b} = 2(1) + (-1)(4) + 2(-1) = -4$$

$$|\mathbf{a}| = \sqrt{4+1+4} = 3, \qquad |\mathbf{b}| = \sqrt{1+16+1} = 3\sqrt{2}$$

$$\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \frac{-4}{9\sqrt{2}} \approx -0.3143$$

$$\theta \approx 108.3^\circ$$

The angle between the vectors is 108° (to the nearest degree).

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Finds $\mathbf{a}\cdot\mathbf{b}$ and both magnitudes, or equivalent merit	1

The scalar product is negative, so the angle is obtuse. Taking the absolute value, or reading 72° off the calculator for $\cos^{-1}(0.3143)$, loses the second mark.

11 (b)

$u = 2x+1$ gives $x = \dfrac{u-1}{2}$ and $dx = \dfrac{du}{2}$. When $x=0$, $u=1$; when $x=4$, $u=9$.

$$\int_0^4 \frac{x}{\sqrt{2x+1}}\,dx = \int_1^9 \frac{\frac{u-1}{2}}{\sqrt{u}}\cdot\frac{du}{2} = \frac{1}{4}\int_1^9 \left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right)du \quad \text{new limits}$$

$$= \frac{1}{4}\left[\frac{2}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\right]_1^9$$

$$= \frac{1}{4}\left[(18-6) - \left(\frac{2}{3}-2\right)\right] = \frac{1}{4}\times\frac{40}{3} = \frac{10}{3}$$

$\dfrac{10}{3}$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Obtains a correct primitive in u with the correct limits, or equivalent merit	2
Writes the integral in terms of u, including $dx = \frac{du}{2}$, or equivalent merit	1

Change the limits when you change the variable. Students who keep 0 and 4 as the u limits get $\frac{1}{4}\left(\frac{16}{3} - 4\right) = \frac{1}{3}$.

11 (c) (i)

Let $X \sim \text{Bin}(8, 0.75)$ be the number of free throws made.

$$P(X = 6) = {}^8C_6(0.75)^6(0.25)^2 = 28 \times 0.177\,98 \times 0.0625 \approx 0.311$$

0.311

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct answer	1

11 (c) (ii)

$$P(X \geq 7) = P(X = 7) + P(X = 8)$$

$$= {}^8C_7(0.75)^7(0.25) + (0.75)^8$$

$$\approx 0.266\,97 + 0.100\,11 = 0.367\,08$$

0.367

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Correctly finds $P(X = 7)$ or $P(X = 8)$, or equivalent merit	1

At least 7 means 7 or 8. Students who find only $P(X = 7) \approx 0.267$ or who find $P(X > 7)$, lose the second mark. Round only at the end.

11 (d)

Let $\theta = \cos^{-1}\left(-\frac{3}{4}\right)$. Then $\cos\theta = -\frac{3}{4}$ and $\frac{\pi}{2} < \theta < \pi$, so $\sin\theta > 0$:

$$\sin\theta = \sqrt{1 - \tfrac{9}{16}} = \frac{\sqrt{7}}{4}, \qquad \tan\theta = \frac{\sin\theta}{\cos\theta} = -\frac{\sqrt{7}}{3} \quad \sin\theta > 0 \text{ in the second quadrant}$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} = \frac{-\frac{2\sqrt{7}}{3}}{1 - \frac{7}{9}} = -\frac{2\sqrt{7}}{3} \times \frac{9}{2} = -3\sqrt{7} \quad \text{double angle}$$

$-3\sqrt{7}$ (Check: $\sin 2\theta = -\frac{3\sqrt{7}}{8}$ and $\cos 2\theta = 2\cos^2\theta - 1 = \frac{1}{8}$, whose quotient is $-3\sqrt{7}$.)

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Finds $\tan\theta = -\frac{\sqrt{7}}{3}$, or $\sin\theta$ and $\cos\theta$ for the angle, or equivalent merit	1

The range of $\cos^{-1}$ is $[0, \pi]$, where sine is never negative, so $\tan\theta$ has the sign of $\cos\theta$. Taking $\tan\theta = +\frac{\sqrt{7}}{3}$ gives $+3\sqrt{7}$, the wrong sign.

11 (e)

$\alpha + \beta + \gamma = 3, \qquad \alpha\beta + \alpha\gamma + \beta\gamma = -6 \quad -\frac{b}{a}$ and $\frac{c}{a}$

$$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$$

$$= 9 - 2(-6) = 21$$

(21) (Check: the roots are 1, -2 and 4, and $1 + 4 + 16 = 21$.)

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Uses $(\alpha + \beta + \gamma)^2 - 2\sum\alpha\beta$ with one value correct, or equivalent merit	1

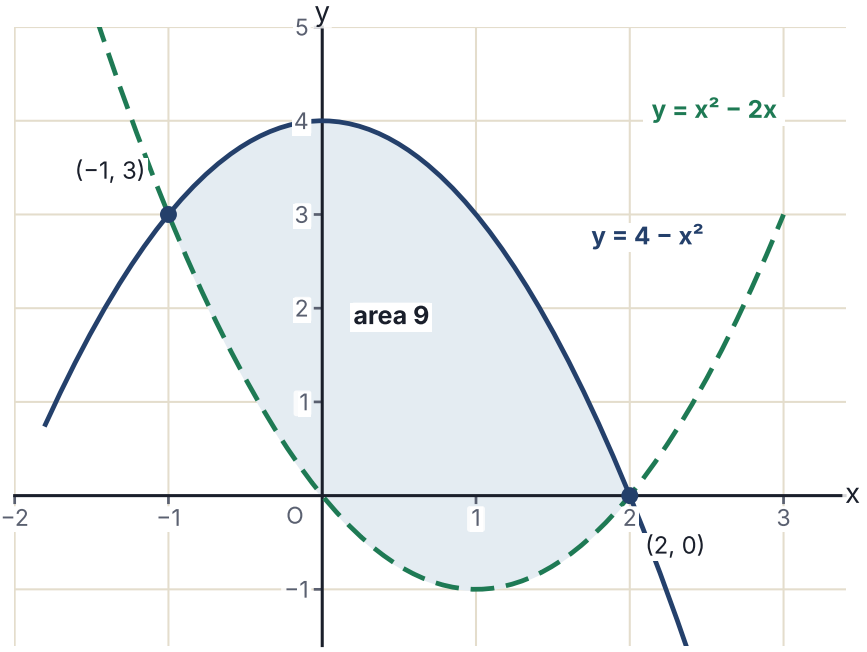
11 (f)

The curves meet where $4 - x^2 = x^2 - 2x$, that is $2x^2 - 2x - 4 = 0$, $(x - 2)(x + 1) = 0$: at $x = -1$ and $x = 2$. Between them $4 - x^2 \geq x^2 - 2x$ (test $x = 0$: $4 > 0$).

$$A = \int_{-1}^{2} \left[(4 - x^2) - (x^2 - 2x)\right] dx = \int_{-1}^{2} \left(4 + 2x - 2x^2\right) dx$$

$$= \left[4x + x^2 - \frac{2x^3}{3}\right]_{-1}^{2} = \left(8 + 4 - \frac{16}{3}\right) - \left(-4 + 1 + \frac{2}{3}\right)$$

$$= \frac{20}{3} + \frac{7}{3} = 9$$



Question 11 (f) · the region between the curves, from $x = -1$ to $x = 2$

The area is 9 square units.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Writes a correct integral for the area with the correct limits, or equivalent merit	2
Finds the x values of the points of intersection, or equivalent merit	1

Part of the region lies below the x-axis, but no splitting is needed: the integral of (upper curve minus lower curve) gives the area wherever the curves are.

Question 12

12 (a)

Let $P(n)$ be the statement $\sum_{r=1}^{n} r(3r-1) = n^2(n+1)$.

Base case, $n = 1$: LHS $= 1 \times 2 = 2$ and RHS $= 1^2 \times 2 = 2$. So $P(1)$ is true.

Inductive step. Assume $P(k)$ is true for some integer $k \geq 1$: $1 \times 2 + 2 \times 5 + \cdots + k(3k-1) = k^2(k+1)$. Prove $P(k+1)$: $1 \times 2 + \cdots + k(3k-1) + (k+1)(3k+2) = (k+1)^2(k+2)$.

$$\text{LHS} = k^2(k+1) + (k+1)(3k+2) \quad \text{by the assumption}$$

$$= (k+1)\left(k^2 + 3k + 2\right) \quad \text{take out } k+1$$

$$= (k+1)(k+1)(k+2) = (k+1)^2(k+2) = \text{RHS}$$

So $P(k+1)$ is true whenever $P(k)$ is true. Since $P(1)$ is true, by mathematical induction $P(n)$ is true for all integers $n \geq 1$.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct proof	3
Proves the base case and uses the assumption to write the left side of $P(k+1)$ correctly, or equivalent merit	2
Proves the base case, or correctly states the assumption and the statement to be proved, or equivalent merit	1

The $(k+1)$th term is $(k+1)\left(3(k+1)-1\right) = (k+1)(3k+2)$. Writing it as $(k+1)(3k-1)$ is the commonest slip. Taking out the common factor $k+1$ at once avoids expanding a cubic.

12 (b) (i)

The velocity of the ball is $\dot{\mathbf{r}}_B = 6\mathbf{i} + 8\mathbf{j}$, so its speed is $\sqrt{36+64} = 10$ m/s.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct answer	1

12 (b) (ii)

The teammate reaches the ball if both components of the two position vectors are equal at the same time.

$$2 + 6t = 14 + 2t \implies t = 3 \quad \mathbf{i} \text{ components}$$

$$1 + 8(3) = 25 \quad \text{and} \quad -2 + 9(3) = 25 \quad \mathbf{j} \text{ components at } t = 3 \text{ agree}$$

At $t = 3$ both are at $2 + 18 = 20$ in the $\mathbf{i}$ direction. The teammate reaches the ball after 3 seconds, at the point with position vector $20\mathbf{i} + 25\mathbf{j}$.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Finds $t = 3$ from one pair of components, or equivalent merit	1

Finding $t = 3$ from the $\mathbf{i}$ components is not enough: the $\mathbf{j}$ components must agree at that same time, otherwise the paths cross at different times and the two never meet.

12 (c)

$$R\sin(x+\alpha) = R\sin x\cos\alpha + R\cos x\sin\alpha$$

$$R\cos\alpha = \sqrt{3}, \quad R\sin\alpha = 1 \implies R = 2,\ \tan\alpha = \tfrac{1}{\sqrt{3}},\ \alpha = \tfrac{\pi}{6}$$

$$2\sin\left(x + \tfrac{\pi}{6}\right) = 1 \implies \sin\left(x + \tfrac{\pi}{6}\right) = \tfrac{1}{2}$$

$$x + \tfrac{\pi}{6} = \tfrac{\pi}{6},\ \tfrac{5\pi}{6},\ \tfrac{13\pi}{6} \qquad \tfrac{\pi}{6} \le x + \tfrac{\pi}{6} \le \tfrac{13\pi}{6}$$

$x = 0, \dfrac{2\pi}{3}, 2\pi$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Obtains $\sin\left(x + \frac{\pi}{6}\right) = \frac{1}{2}$ and finds two correct solutions, or equivalent merit	2
Finds $R = 2$ and $\alpha = \frac{\pi}{6}$, or equivalent merit	1

Shift the interval along with the angle: $x + \frac{\pi}{6}$ runs up to $\frac{13\pi}{6}$, which brings in the solution $x = 2\pi$. Stopping at 2π for $x + \frac{\pi}{6}$ loses it.

12 (d)

$$\int_0^2 \frac{dx}{12 + 3x^2} = \frac{1}{3}\int_0^2 \frac{dx}{4 + x^2} = \frac{1}{3} \times \frac{1}{2}\int_0^2 \frac{2}{2^2 + x^2}\,dx \quad \text{form } \tfrac{a}{a^2+x^2} \text{ with } a = 2$$

$$= \frac{1}{6}\left[\tan^{-1}\frac{x}{2}\right]_0^2 = \frac{1}{6}\left(\frac{\pi}{4} - 0\right)$$

$\dfrac{\pi}{24}$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Obtains a primitive of the form $k\tan^{-1}\frac{x}{2}$, or equivalent merit	1

The reference sheet gives $\int \frac{a}{a^2+x^2}\,dx$, with a on top. Without the factor $\frac{1}{2}$ the answer comes out as $\frac{\pi}{12}$.

12 (e)

$f'(x) = 5x^4 + 3x^2$. Since $f(1) = 3$, $f^{-1}(3) = 1$, and

$$\left(f^{-1}\right)'(3) = \frac{1}{f'\left(f^{-1}(3)\right)} = \frac{1}{f'(1)} = \frac{1}{5+3} = \frac{1}{8}$$

The gradient of the tangent is $\dfrac{1}{8}$.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Uses $\left(f^{-1}\right)'(x) = \dfrac{1}{f'\left(f^{-1}(x)\right)}$ or $\frac{dx}{dy} = 1 \div \frac{dy}{dx}$, or equivalent merit	1

Evaluate f' at 1, not at 3: $\frac{1}{f'(3)} = \frac{1}{432}$ is the usual wrong answer. The tangents at $(1, 3)$ and $(3, 1)$ are reflections in $y = x$, so their gradients are reciprocals.

12 (f)

Count all squads and subtract those with 0 or 1 defender:

$^{12}C_6 = 924$ all squads

$^{7}C_6 + {}^{5}C_1 \times {}^{7}C_5 = 7 + 5 \times 21 = 112$ no defender, one defender

$924 - 112 = 812$

$\boxed{812}$ squads.

MARKING GUIDELINES	
CRITERIA	MARKS
Provides correct solution	2
Finds the number of squads with no defender or with exactly one defender, or equivalent merit	1

Choosing 2 defenders first and then any 4 others, $^{5}C_2 \times {}^{10}C_4 = 2100$, counts squads with 3 or more defenders several times. Use the complement or add the cases $2, 3, 4, 5$ defenders.

Question 13

13 (a)

$x \neq 2$. Multiply both sides by $(x-2)^2$, which is positive:

$(x+5)(x-2) \leq 3(x-2)^2$

$(x-2)\left[(x+5) - 3(x-2)\right] \leq 0$ collect on one side and factorise

$(x-2)(11-2x) \leq 0$

$x \leq 2$ or $x \geq \frac{11}{2}$, $\quad x \neq 2$ a parabola opening downwards in x

$\boxed{x < 2 \text{ or } x \geq \dfrac{11}{2}}$

MARKING GUIDELINES	
CRITERIA	MARKS
Provides correct solution	3
Obtains $(x-2)(11-2x) \leq 0$, or equivalent merit	2
Multiplies by $(x-2)^2$, or finds the critical values 2 and $\frac{11}{2}$, or equivalent merit	1

Multiplying by $x-2$ without knowing its sign gives $x+5 \leq 3x-6$ so $x \geq \frac{11}{2}$, which loses every $x < 2$. Test $x = 0$: $\frac{5}{-2} = -2.5$, which is ≤ 3.

13 (b)

$$T_{r+1} = {}^7C_r(2x)^{7-r}\left(-\frac{3}{x}\right)^r = {}^7C_r\,2^{7-r}(-3)^r x^{7-2r} \quad \text{general term}$$

$$7 - 2r = 5 \implies r = 1$$

$${}^7C_1 \times 2^6 \times (-3) = 7 \times 64 \times (-3) = -1344$$

-1344

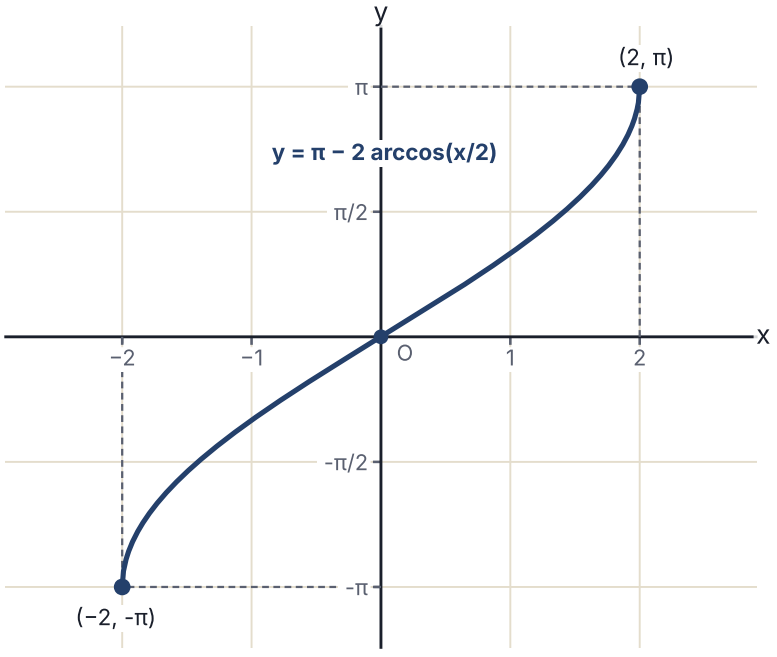
MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Finds the general term and $r = 1$, or equivalent merit	1

The coefficient includes the power of 2. Answers of -21 (forgetting 2^6) and 1344 (losing the sign of -3) are both common.

13 (c)

- **Domain.** $\cos^{-1}\left(\frac{x}{2}\right)$ needs $-1 \le \frac{x}{2} \le 1$: $-2 \le x \le 2$.
- **Range.** $0 \le \cos^{-1}\left(\frac{x}{2}\right) \le \pi$, so $0 \le 2\cos^{-1}\left(\frac{x}{2}\right) \le 2\pi$ and $-\pi \le y \le \pi$ $-\pi \le y \le \pi$.
- **Endpoints.** $x = -2$: $y = \pi - 2\pi = -\pi$ $x = 2$ $y = \pi - 0 = \pi$. So $(-2, -\pi)$ and $(2, \pi)$.
- **Intercepts.** $x = 0$: $y = \pi - 2 \times \frac{\pi}{2} = 0$, and $y = 0$ only when $\cos^{-1}\left(\frac{x}{2}\right) = \frac{\pi}{2}$, that is $x = 0$. The graph passes through the origin.
- **Shape.** $\cos^{-1}$ is decreasing, so y is increasing. In fact $\pi - 2\cos^{-1}\left(\frac{x}{2}\right) = 2\left(\frac{\pi}{2} - \cos^{-1}\frac{x}{2}\right) = 2\sin^{-1}\left(\frac{x}{2}\right)$, an odd function, least steep at the
- origin and vertical at the endpoints.



Question 13 (c) · $y = \pi - 2\cos^{-1}\left(\frac{x}{2}\right)$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct sketch	3
Correct shape with two of: the domain, the range or endpoints, the intercept at the origin, or equivalent merit	2
Finds the domain or the range, or equivalent merit	1

Do each transformation on the domain and range in turn, as NESA advises: $\frac{x}{2}$ stretches the domain to $[-2, 2]$, and $\pi - 2(\ldots)$ reflects, stretches and shifts the range.

13 (d) (i)

$T = 24 - 20e^{-kt}$ gives $\dfrac{dT}{dt} = 20ke^{-kt}$, and $-k(T - 24) = -k\left(-20e^{-kt}\right) = 20ke^{-kt}$. These are equal, so the differential equation is satisfied. At $t = 0$, $T = 24 - 20 = 4$, the temperature leaving the fridge.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	1

13 (d) (ii)

$$9 = 24 - 20e^{-10k} \implies e^{-10k} = \tfrac{3}{4} \implies k = \tfrac{1}{10}\ln\tfrac{4}{3} \approx 0.028\,77 \quad t = 10$$

$$18 = 24 - 20e^{-kt} \implies e^{-kt} = \tfrac{3}{10} \quad T = 18$$

$$t = \frac{\ln\frac{10}{3}}{k} = \frac{10\ln\frac{10}{3}}{\ln\frac{4}{3}} \approx 41.85$$

The drink reaches $18°$C after about 42 minutes.

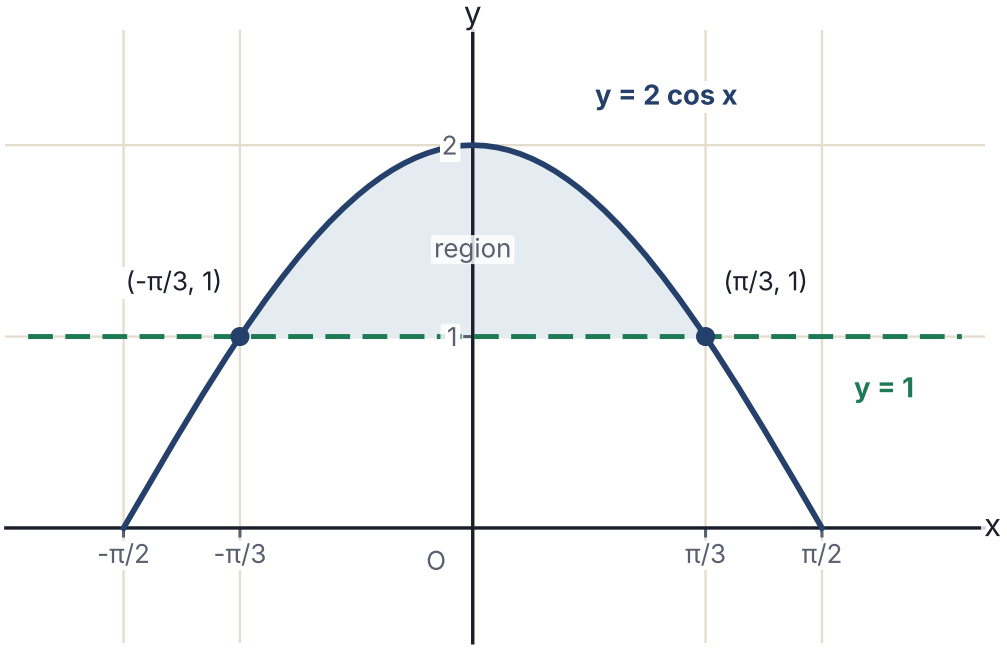
MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Finds k and obtains $e^{-kt} = \frac{3}{10}$, or equivalent merit	2
Finds $k = \frac{1}{10}\ln\frac{4}{3}$, or equivalent merit	1

Keep k exact (or in the calculator memory). Rounding k to 0.03 gives 40 minutes, which is wrong to the nearest minute.

13 (e)

On $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$, $2\cos x \ge 1$. Rotating the region gives a solid with outer radius $2\cos x$ and inner radius 1, so subtract the cylinder:



Question 13 (e) · the region between $y = 2\cos x$ and $y = 1$

$$V = \pi \int_{-\pi/3}^{\pi/3} \left(4\cos^2 x - 1\right) dx$$

$$= \pi \int_{-\pi/3}^{\pi/3} (2(1 + \cos 2x) - 1)\, dx = \pi \int_{-\pi/3}^{\pi/3} (1 + 2\cos 2x)\, dx \quad \cos^2 x = \tfrac{1}{2}(1 + \cos 2x)$$

$$= \pi \left[x + \sin 2x\right]_{-\pi/3}^{\pi/3} = \pi \left[\left(\tfrac{\pi}{3} + \tfrac{\sqrt{3}}{2}\right) - \left(-\tfrac{\pi}{3} - \tfrac{\sqrt{3}}{2}\right)\right]$$

$$= \pi \left(\frac{2\pi}{3} + \sqrt{3}\right)$$

$V = \pi \left(\dfrac{2\pi}{3} + \sqrt{3}\right)$ cubic units (about 12.02).

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Uses $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ in a correct volume integral, or equivalent merit	2
Writes $V = \pi \int \left[(2\cos x)^2 - 1^2\right] dx$ with correct limits, or equivalent merit	1

The volume is $\pi \int \left(y_1^2 - y_2^2\right) dx$, not $\pi \int (y_1 - y_2)^2\, dx$: subtract the squares, because the hollow is a whole cylinder of radius 1.

Question 14

14 (a)

The range of f is $y \geq 3$ which becomes the domain of f^{-1}. Swap x and y:

$$x = 2(y-1)^2 + 3, \quad y \geq 1$$

$$(y-1)^2 = \frac{x-3}{2}$$

$$y - 1 = \sqrt{\frac{x-3}{2}} \quad \text{positive root, because } y \geq 1$$

$$f^{-1}(x) = 1 + \sqrt{\frac{x-3}{2}}, \text{ domain } x \geq 3$$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Finds $f^{-1}(x)$, or states the domain, or equivalent merit	1

14 (b) (i)

The sample size $n = 36 \geq 30$ so by the central limit theorem $\overline{X}$ is approximately normal even though the population is skewed. $\mathrm{E}(\overline{X}) = 31$ and the standard deviation is $\dfrac{7.2}{\sqrt{36}} = 1.2$ minutes.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct explanation and values	1

14 (b) (ii)

$$z = \frac{29.2 - 31}{1.2} = -1.5$$

$$P(\overline{X} < 29.2) = P(Z < -1.5) = 1 - P(Z \leq 1.5) = 1 - 0.9332 = 0.0668 \quad \text{symmetry}$$

0.0668

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Finds $z = -1.5$ using the standard deviation 1.2, or equivalent merit	1

Use $\frac{\sigma}{\sqrt{n}} = 1.2$, not $\sigma = 7.2$: with 7.2, $z = -0.25$, and the probability is wrongly about 0.40.

14 (c)

In the right triangle NCP, $\tan\theta = \dfrac{x}{30}$, so $\theta = \tan^{-1}\dfrac{x}{30}$.

$$\frac{d\theta}{dx} = \frac{\frac{1}{30}}{1+\frac{x^2}{900}} = \frac{30}{900+x^2}$$

$$\frac{d\theta}{dt} = \frac{d\theta}{dx} \times \frac{dx}{dt} = \frac{30}{900+x^2} \times 16 \quad \text{chain rule}$$

$$= \frac{480}{900+1600} = 0.192 \quad x = 40$$

The angle is increasing at 0.192 radians per second.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Obtains $\frac{d\theta}{dt} = \frac{30}{900+x^2} \times 16$, or equivalent merit	2
Writes $\tan\theta = \frac{x}{30}$ and finds $\frac{d\theta}{dx}$, or equivalent merit	1

Substitute $x = 40$ only after differentiating. Putting $x = 40$ in first freezes the angle at a constant and gives a rate of 0.

14 (d) (i)

Integrate the acceleration, using $\mathbf{v}(0) = 20\cos\theta\,\mathbf{i} + 20\sin\theta\,\mathbf{j}$ and $\mathbf{r}(0) = \mathbf{0}$:

$$\mathbf{v}(t) = c_1\mathbf{i} + (-10t + c_2)\mathbf{j} \quad \text{integrate } \mathbf{a} = -10\mathbf{j}$$

$$\mathbf{v}(t) = 20\cos\theta\,\mathbf{i} + (20\sin\theta - 10t)\mathbf{j} \quad t = 0\text{: } c_1 = 20\cos\theta, c_2 = 20\sin\theta$$

$$\mathbf{r}(t) = (20t\cos\theta + c_3)\mathbf{i} + \left(20t\sin\theta - 5t^2 + c_4\right)\mathbf{j} \quad \text{integrate again}$$

$$\mathbf{r}(t) = (20t\cos\theta)\mathbf{i} + \left(20t\sin\theta - 5t^2\right)\mathbf{j} \quad t = 0\text{: } c_3 = c_4 = 0$$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct derivation	2
Finds $\mathbf{v}(t)$ using the initial velocity, or equivalent merit	1

14 (d) (ii)

From $x = 20t\cos\theta, t = \dfrac{x}{20\cos\theta}$. Substitute into y:

$$y = 20\sin\theta \cdot \frac{x}{20\cos\theta} - 5 \cdot \frac{x^2}{400\cos^2\theta} = x\tan\theta - \frac{x^2\sec^2\theta}{80}$$

$$y = x\tan\theta - \frac{x^2\left(1+\tan^2\theta\right)}{80} \quad \sec^2\theta = 1+\tan^2\theta$$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct derivation	1

14 (d) (iii)

The ball passes above the crossbar when $y > 3$ at $x = 30$. Let $T = \tan\theta$, which is positive:

$$30T - \frac{900\left(1+T^2\right)}{80} > 3$$

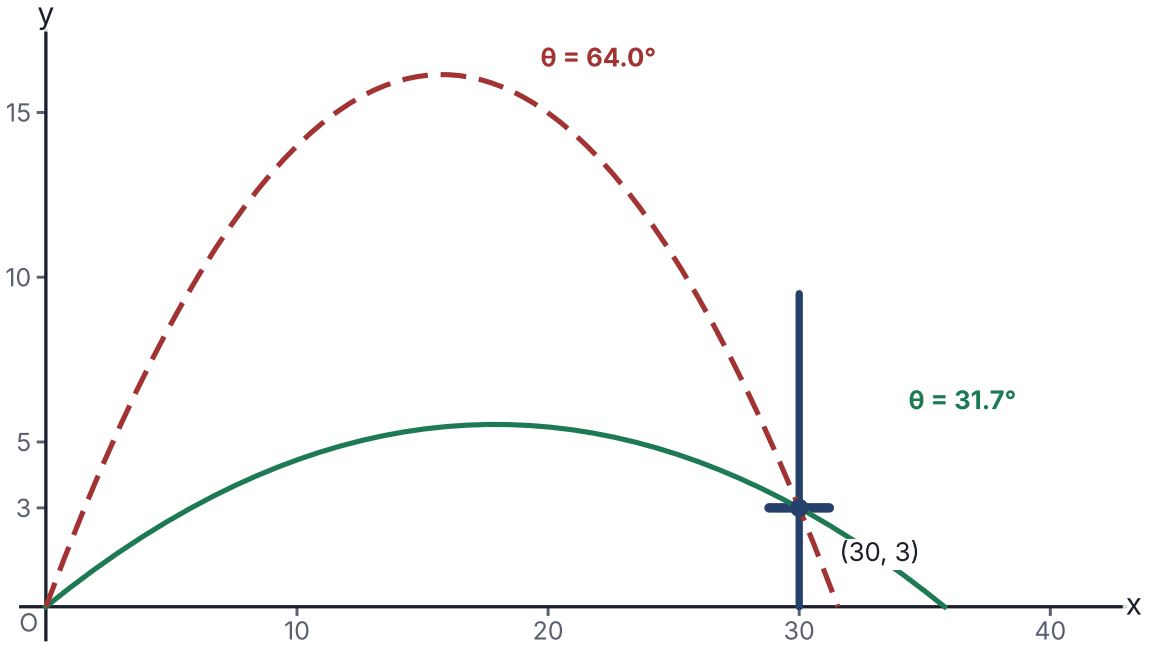
$$30T - \frac{45}{4}\left(1+T^2\right) > 3 \implies 45T^2 - 120T + 57 < 0 \quad \times 4, \text{ rearrange}$$

$$15T^2 - 40T + 19 < 0 \quad \div 3$$

$$T = \frac{40 \pm \sqrt{1600-1140}}{30} = \frac{40 \pm \sqrt{460}}{30} \quad \text{boundary values}$$

$$0.6184 < \tan\theta < 2.0483 \quad \text{between the roots}$$

$\tan$ is increasing on $0 < \theta < \frac{\pi}{2}$, so the kick passes above the crossbar for $31.7^\circ < \theta < 64.0^\circ$ (correct to one decimal place).



Question 14 (d) · the two boundary kicks pass exactly through the top of the crossbar, $(30, 3)$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	4
Solves the quadratic inequality in $\tan\theta$ correctly, or equivalent merit	3
Obtains $15\tan^2\theta - 40\tan\theta + 19 < 0$, or equivalent merit	2
Substitutes $x = 30$ and $y = 3$ into the Cartesian equation, or equivalent merit	1

The answer is an interval, not two angles. A low kick hits the bar on the way up; a very high kick has come back down below 3 m before it reaches the posts. Both boundary angles matter, and rounding them to whole degrees $(32^\circ, 64^\circ)$ would include 64°, a kick that just fails.

What this paper examined

Focus area	Section I	Section II	Marks	Target
Proof by mathematical induction (ME1-12-01)	1	3	**4**	4
Introduction to vectors (ME1-12-02)	2	12	**14**	14
Inverse trigonometric functions (ME1-12-03)	2	5	**7**	7
Further calculus skills (ME1-12-04)	2	7	**9**	9
Further applications of calculus (ME1-12-05)	1	13	**14**	14
The binomial distribution and sampling distribution of the mean (ME1-12-06)	1	6	**7**	7
Year 11 course (assumed knowledge) (ME1-11-01 to 05)	1	14	**15**	15
Total	**10**	**60**	**70**	**70**

The targets follow the HSC pattern: about a fifth of the marks on the Year 11 course, and the rest shared across the six Year 12 focus areas in proportion to their size. Where each was examined:

- **Proof by mathematical induction:** Question 9 (the inductive step of a divisibility proof); 12 (a) a sum.
- **Introduction to vectors:** Questions 1 and 7 (3D magnitude, perpendicular vectors); 11 (a) the angle between 3D vectors; 12 (b) constant velocity and meeting; 14 (d) projectile motion from the acceleration vector to the Cartesian path and an unknown angle of projection.
- **Inverse trigonometric functions:** Questions 4 and 10 (domain and range, a composite); 11 (d) an exact value; 13 (c) a transformed graph.
- **Further calculus skills:** Questions 2 and 8 ($\cos^2 nx$, parametric derivative); 11 (b) a given substitution; 12 (d) the inverse tangent integral; 12 (e) the derivative of an inverse function.
- **Further applications of calculus:** Question 6; 11 (f) area between curves; 13 (d) Newton's law of cooling (here warming); 13 (e) a volume of revolution; 14 (c) related rates.
- **The binomial distribution and sampling distribution of the mean:** Question 5; 11 (c) binomial probabilities; 14 (b) the central limit theorem.
- **Year 11 course:** Question 3; 11 (e) sums and products of roots; 12 (c) the auxiliary angle; 12 (f) selections; 13 (a) a rational inequality; 13 (b) the binomial theorem; 14 (a) an inverse function.

Use the table to find the focus area where you lost the most marks, and work through that focus area's Practice Questions and Exemplar Response before you sit Practice Paper 2.

End of worked solutions.

About this resource

MA99 EDUCATION

Scope

NSW Mathematics Extension 1, Year 12. Written to the NSW Mathematics Extension 1 11-12 Syllabus (2024), an HSC style paper on the whole course: the Year 12 focus areas, with the Year 11 course as assumed knowledge, for students sitting the HSC in 2027 and later years. Every value and solution was re-derived independently by computer algebra before printing. Contexts in the questions are invented unless a source is named.

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