MA99 EDUCATION

Mathematics Advanced: Practice Paper 1

01

NSW MATHEMATICS ADVANCED 11-12 SYLLABUS 2024

FOR HSC 2027 AND LATER

General Instructions

- Reading time: 10 minutes
- Working time: 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- The NESA Mathematics Advanced reference sheet may be used.
- From Question 11 on, show your mathematical reasoning and/or calculations in every answer

Total marks: 100

Section I · 10 marks

Questions 1 to 10

Suggested time: about 15 minutes

Section II · 90 marks

Questions 11 to the end

Suggested time: about 2 hours 45 minutes

Year 11 work is assumed, and questions can draw on it.

This paper examines the whole Year 12 Mathematics Advanced course of the 2024 syllabus (further graph transformations and modelling, sequences and series, differential calculus, integral calculus, applications of calculus, random variables and financial mathematics), with about a fifth of the marks on the Year 11 course. Questions with a setting are drawn from sport, fitness and recreation. Formulas that are not on the reference sheet are given in the question that needs them.

Marks may be deducted for careless or badly arranged work.

Section I · Answer Sheet

Shade ONE circle for each of Questions 1 to 10. If you change your mind, draw a cross through the incorrect answer and shade the new one.

1	A	B	C	D	2	A	B	C	D
3	A	B	C	D	4	A	B	C	D
5	A	B	C	D	6	A	B	C	D
7	A	B	C	D	8	A	B	C	D
9	A	B	C	D	10	A	B	C	D

Section I · 10 marks

Answer Questions 1 to 10 on the multiple choice answer sheet, spending about 15 minutes on this section.

Question 1

1 MARK

Which expression is equal to $\dfrac{6}{\sqrt{5}+\sqrt{2}}$?

A $2\left(\sqrt{5}+\sqrt{2}\right)$

B $2\left(\sqrt{5}-\sqrt{2}\right)$

C $\dfrac{6\left(\sqrt{5}-\sqrt{2}\right)}{7}$

D $\dfrac{\sqrt{5}-\sqrt{2}}{3}$

Question 2

1 MARK

Training for a fun run, Priya runs 3 km on the first day and increases her distance by 0.5 km each day after that. What total distance does she run over the first 14 days?

A 9.5 km

B 87.5 km

C 91 km

D 175 km

Question 3

1 MARK

What is the derivative of $x \ln x$?

A 1

B $\ln x$

C $\ln x + 1$

D $\dfrac{1}{x}$

Question 4

1 MARK

What is the value of $\displaystyle\int_0^{\frac{\pi}{6}} \cos 2x \, dx$?

A $\dfrac{\sqrt{3}}{4}$

B $\dfrac{\sqrt{3}}{2}$

C $\sqrt{3}$

D $\dfrac{1}{4}$

Question 5

1 MARK

The number of goals, X, that a hockey team scores in a match has the probability distribution shown, where k is a constant. What is $\mathrm{E}(X)$?

x	0	1	2	3
$P(X = x)$	0.1	k	$2k$	0.3

A 1.5

B 0.475

C 2.4

D 1.9

Question 6

1 MARK

Events A and B are such that $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cup B) = 0.7$. What is $P(A \mid B)$?

A 0.2

B 0.3

C 0.4

D 0.5

Question 7

1 MARK

What are the period and the range of $y = 2\sin\left(\dfrac{\pi x}{6}\right) - 1$?

A Period 12, range $-3 \leq y \leq 1$

B Period 12, range $-2 \leq y \leq 2$

C Period $\dfrac{\pi^2}{3}$, range $-3 \leq y \leq 1$

D Period 12, range $-1 \leq y \leq 3$

Question 8

1 MARK

A gym owner borrows $\$P$ at an interest rate of r per month, compounded monthly. The loan is repaid with equal repayments of $\$M$ at the end of each month. Which expression is the amount owing, in dollars, immediately after the second repayment?

A $P(1+r)^2 - 2M$

B $(P - M)(1+r)^2$

C $P(1+r)^2 - M(1+r) - M$

D $P(1+r)^2 - M(1+r)^2 - M(1+r)$

Question 9

1 MARK

The time, in hours, that a cyclist spends on a training ride is a continuous random variable X with cumulative distribution function $F(x) = \dfrac{x^2 + x}{6}$ for $0 \leq x \leq 2$What is the median of X?

A 1

B $\dfrac{\sqrt{13} - 1}{2}$

C $\dfrac{11}{9}$

D $\dfrac{\sqrt{7} - 1}{2}$

Question 10

1 MARK

Let $f(x) = \left|x^2 - 2x\right|$. For which values of x is f continuous but not differentiable?

A $x = 0$ and $x = 2$ only

B $x = 1$ only

C $x = 0$, $x = 1$ and $x = 2$

D There are no such values

Section II · 90 marks

Answer Questions 11 to 37 in the spaces provided, spending about 2 hours 45 minutes on this section. Show your mathematical reasoning and/or calculations in every answer.

Question 11 · 2 marks

Simplify $\dfrac{x^2-9}{x^2+x-6} \div \dfrac{x-3}{2x-4}$.

Question 12 · 2 marks

Differentiate $y = x^2 e^{3x}$.

Question 13 · 2 marks

Find the value of $\sum_{k=5}^{24} (3k-2)$.

Question 14 · 5 marks

(a) Find $\int \dfrac{6x}{x^2+4}\,dx$. 2 MARKS

(b) Evaluate $\int_1^4 \frac{x+2}{\sqrt{x}}\,dx$. 3 MARKS

Question 15 · 3 marks

Solve $2\cos^2 x + 3\cos x + 1 = 0$ for $0 \leq x \leq 2\pi$.

Question 16 · 2 marks

Find the centre and the radius of the circle $x^2 + y^2 - 10x + 6y + 9 = 0$.

Question 17 · 5 marks

The temperature of the water in an outdoor swimming pool, T degrees Celsius, t hours after midnight on a summer day is modelled by

$$T = 24 - 3\cos\left(\frac{\pi t}{12}\right), \qquad 0 \le t \le 24.$$

(a) Sketch the graph of T for $0 \le t \le 24$, showing the maximum and minimum temperatures and the times when they occur. 2 MARKS

(b) The pool's heating switches off whenever the water temperature is at least 25.5 degrees. Between what times of the day is the heating switched off? 3 MARKS

Question 18 · 4 marks

A netball team plays 60% of its games at home. The probability that the team wins a home game is 0.7, and the probability that it wins an away game is 0.4.

(a) Find the probability that the team wins a game chosen at random. 2 MARKS

(b) A game chosen at random was won. Find the probability that it was a home game. 2 MARKS

Question 19 · 4 marks

An archer's score on one arrow is a discrete random variable X with the probability distribution shown.

x	0	5	7	9	10
$P(X = x)$	0.1	0.2	p	0.3	0.1

(a) Find the value of p and $\mathrm{E}(X)$. 2 MARKS

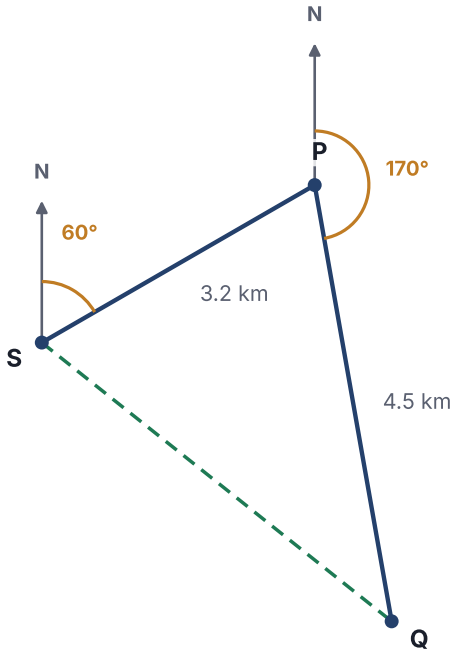
(b) Find $\mathrm{Var}(X)$. 2 MARKS

Question 20 · 2 marks

The sound level, L decibels (dB), of a sound of intensity I is given by $L = 10\log_{10}\left(\frac{I}{I_0}\right)$, where I_0 is a fixed reference intensity. During a grand final the crowd noise rose from 88 dB to 106 dB. How many times as intense was the louder crowd noise? Give your answer correct to the nearest whole number.

Question 21 · 4 marks

In an orienteering event, Jack runs 3.2 km from the start S on a bearing of 060° to a checkpoint P. He then runs 4.5 km on a bearing of 170° to a checkpoint Q, as shown.



Question 21 · the orienteering course (not to scale)

Find the distance SQ, correct to two decimal places, and the bearing of S from Q, correct to the nearest degree.

Question 22 · 3 marks

Find the equation of the tangent to the curve $y = \ln(2x - 1)$ at the point where the curve crosses the x-axis.

Question 23 · 4 marks

A sprinter's velocity, v metres per second, t seconds after the start of a race is modelled by $v = 12\left(1 - e^{-0.8t}\right)$. The sprinter starts from rest at the starting line.

(a) Find the sprinter's acceleration at the start of the race. 1 MARK

(b) Find the distance the sprinter runs in the first 10 seconds, correct to the nearest metre. 3 MARKS

Question 24 · 3 marks

A cyclist rides 60 km in the first week of her training program. Each week after that she rides 10% further than in the week before.

(a) In which week does she first ride more than 150 km? 2 MARKS

(b) Find the total distance she rides over the first 11 weeks, correct to the nearest kilometre. 1 MARK

Question 25 · 3 marks

(a) Evaluate $\int_0^3 \left(x^2 - 2x\right) dx$. 1 MARK

(b) Find the area of the region bounded by the curve $y = x^2 - 2x$, the x-axis and the lines $x = 0$ and $x = 3$. 2 MARKS

Question 26 · 2 marks

A rowing coach records the speed of a boat every 10 seconds during the first 40 seconds of a race.

Time t (s)	0	10	20	30	40
Speed v (m/s)	0	3.2	4.1	4.5	4.6

Use the trapezoidal rule with these five speeds to estimate the distance, in metres, that the boat travels in the first 40 seconds.

Question 27 · 2 marks

Solve $\log_2 x + \log_2(x+2) = 3$.

Question 28 · 6 marks

A rowing club deposits $\$5000$ into an account at the end of every year. The account earns interest at 4% per annum, compounded annually.

(a) Show that the amount in the account immediately after the nth deposit is $\$125\,000\,(1.04^n - 1)$. 2 MARKS

(b) After how many deposits will the amount in the account first exceed $\$80\,000$? 2 MARKS

(c) The club wants exactly $\$80\,000$ in the account immediately after the 10th deposit. What equal annual deposit is needed? Give your answer to the nearest cent. 2 MARKS

Question 29 · 7 marks

Let $f(x) = x^4 + 2x^3$.

(a) Find the stationary points of $y = f(x)$ and determine their nature. 3 MARKS

(b) Find the points of inflection of $y = f(x)$. 2 MARKS

(c) Sketch the graph of $y = f(x)$, showing the stationary points, the points of inflection and the intercepts with the axes. 2 MARKS

Question 30 · 3 marks

Find the coordinates of the points on the curve $y = x^3 - 3x^2 + 1$ at which the tangent is parallel to the line $y = 9x + 4$.

Question 31 · 3 marks

The time, T minutes, that a member of a gym waits for a free treadmill is a continuous random variable with probability density function

$$f(t) = \begin{cases} k\left(5t - t^2\right), & 0 \leq t \leq 5 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Show that $k = \dfrac{6}{125}$. 1 MARK

(b) Find the probability that a member waits no more than 1 minute. 2 MARKS

Question 32 · 2 marks

Find the value of k, where $k > 1$, such that $\int_1^k (2x - 1)\,dx = 6$.

Question 33 · 3 marks

Let $y = e^{2x}\cos x$. Show that $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 5y = 0$.

Question 34 · 4 marks

The finishing times in a 5 km fun run are normally distributed with a mean of 28 minutes and a standard deviation of 4 minutes.

(a) Use the empirical rule to find the percentage of runners who finish between 24 and 36 minutes. 2 MARKS

(b) Mia finished the fun run in 25 minutes. In a 100 m swimming race, where the times were normally distributed with a mean of 56 seconds and a standard deviation of 2.5 seconds, she swam 52 seconds. Relative to the other competitors, in which event did Mia perform better? Justify your answer using z-scores. 2 MARKS

Question 35 · 2 marks

The gradient function of a curve is $f'(x) = 2\cos 2x$, and the curve passes through the point $\left(\frac{\pi}{4}, 3\right)$. Find the equation of the curve.

Question 36 · 3 marks

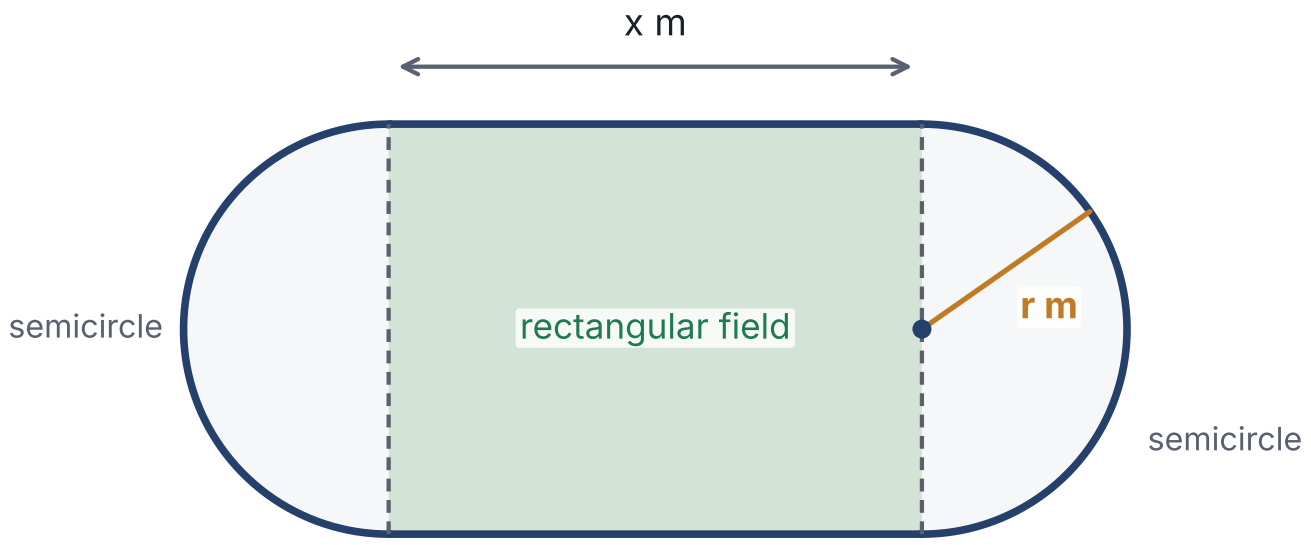
Find the value of x for which the geometric series

$$1 + \frac{x-1}{2} + \left(\frac{x-1}{2}\right)^2 + \left(\frac{x-1}{2}\right)^3 + \cdots$$

has a limiting sum of 4. Justify that the series has a limiting sum for this value of x.

Question 37 · 5 marks

The inside edge of an athletics track is made of two straight sections, each x metres long, joined by two semicircles of radius r metres. The total length of the inside edge is 400 metres. A rectangular field fills the region between the two straight sections, as shown.



Question 37 · the inside edge of the track and the rectangular field

(a) Show that $A = 400r - 2\pi r^2$, where A square metres is the area of the rectangular field. 1 MARK

(b) Find the dimensions of the field that give the greatest possible area, and find this area correct to the nearest square metre. Justify that your answer gives the greatest area. 4 MARKS

End of paper.

About this resource

MA99 EDUCATION

Scope

NSW Mathematics Advanced, Year 12. Written to the NSW Mathematics Advanced 11-12 Syllabus (2024), an HSC style paper on the whole course: the Year 12 focus areas, with the Year 11 course as assumed knowledge, for students sitting the HSC in 2027 and later years. Every value and solution was re-derived independently by computer algebra before printing. Contexts in the questions are invented unless a source is named.

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MA99 EDUCATION

Mathematics Advanced: Practice Paper 1 · Worked Solutions

01

NSW MATHEMATICS ADVANCED 11-12 SYLLABUS 2024

FOR HSC 2027 AND LATER

Worked solutions · Section I

1

Multiply the numerator and the denominator by the conjugate $\sqrt{5}-\sqrt{2}$:

$$\frac{6}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{6\left(\sqrt{5}-\sqrt{2}\right)}{5-2} = 2\left(\sqrt{5}-\sqrt{2}\right)$$

(B)

(A) multiplies the top and bottom by $\sqrt{5}+\sqrt{2}$ instead of the conjugate, then simplifies the denominator $\left(\sqrt{5}+\sqrt{2}\right)^2$ as if it were $5-2$. (C) writes the denominator as $5+2=7$: the product of conjugates is a difference of squares. (D) loses the numerator 6 when multiplying by the conjugate: it is $\dfrac{\sqrt{5}-\sqrt{2}}{5-2}$.

2

The daily distances form an AP with $a=3$, $d=0.5$ and $n=14$:

$$S_{14} = \frac{14}{2}\left[2(3)+13(0.5)\right] = 7 \times 12.5 = 87.5$$

(B)

(A) 9.5 km is $a_{14} = 3+13(0.5)$, the distance on the 14th day alone, not the total. (C) 91 km uses nd instead of $(n-1)d$: $7[6+14(0.5)]=91$. (D) 175 km leaves out the $\frac{1}{2}$: 14×12.5.

3

Product rule with $u=x$ and $v=\ln x$: $\dfrac{d}{dx}(x\ln x) = 1 \times \ln x + x \times \dfrac{1}{x} = \ln x + 1$.

(C)

(A) multiplies the two derivatives, $1 \times \dfrac{1}{x}$, and then cancels with x: the product of derivatives is not the derivative of a product. (B) keeps only the term $v\,u'$. (D) $\dfrac{1}{x}$ is the derivative of $\ln x$ alone, treating x as a constant factor.

4

$$\int_0^{\frac{\pi}{6}} \cos 2x\,dx = \left[\tfrac{1}{2}\sin 2x\right]_0^{\frac{\pi}{6}} = \tfrac{1}{2}\sin\tfrac{\pi}{3} - 0 = \tfrac{1}{2} \times \tfrac{\sqrt{3}}{2} = \tfrac{\sqrt{3}}{4}$$

(A)

(B) forgets the factor $\frac{1}{2}$ in the primitive $\frac{1}{2}\sin 2x$. (C) multiplies by 2 instead of dividing (the chain rule run forwards, as for a derivative). (D) evaluates $\frac{1}{2}\sin x$ at $\frac{\pi}{6}$, dropping the 2 inside the sine when substituting.

5

The probabilities add to 1: $0.1+k+2k+0.3=1$, so $3k=0.6$ and $k=0.2$.

$$\mathrm{E}(X) = 0(0.1)+1(0.2)+2(0.4)+3(0.3) = 0+0.2+0.8+0.9 = 1.9$$

(D)

(A) 1.5 is the plain average of $0, 1, 2, 3$, as if every value were equally likely. (B) divides the correct 1.9 by 4, the number of values: $\mathrm{E}(X)$ is a weighted sum, not divided again. (C) 2.4 comes from $2k + 0.4 = 1$, missing the term k, so $k = 0.3$ and $\mathrm{E}(X) = 0.3 + 1.2 + 0.9$.

6

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.5 + 0.4 - 0.7 = 0.2 \quad \text{addition rule}$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.4} = 0.5$$

(D)

(A) 0.2 is $P(A \cap B)$, stopped one step early. (B) $0.3 = 0.7 - 0.4$ is the probability of A without B, not of A given B. (C) $0.4 = \dfrac{0.2}{0.5}$ divides by $P(A)$: it is $P(B \mid A)$. (Since $P(A \mid B) = P(A)$, the events happen to be independent.)

7

Period $= \dfrac{2\pi}{\pi/6} = 12$. The amplitude is 2 and the graph is moved down 1, so $-2 - 1 \leq y \leq 2 - 1$that is $-3 \leq y \leq 1$.

(A)

(B) ignores the vertical translation of -1. (C) multiplies instead of dividing: $2\pi \times \frac{\pi}{6} = \frac{\pi^2}{3}$. (D) moves the graph up 1 instead of down.

8

Each month the balance earns interest and then a repayment is subtracted:

$$A_1 = P(1 + r) - M \quad \text{after month 1}$$

$$A_2 = A_1(1 + r) - M = P(1 + r)^2 - M(1 + r) - M \quad \text{after month 2}$$

(C)

(A) charges month 2 interest on the full $P(1 + r)$, ignoring that the first repayment has already reduced the balance: that repayment saves a month of interest, so it is worth $M(1 + r)$ by the end of month 2. (B) takes the first repayment off before any interest is charged and never subtracts the second. (D) compounds each repayment for one month too many: the second repayment is made at the end of month 2 and saves no interest.

9

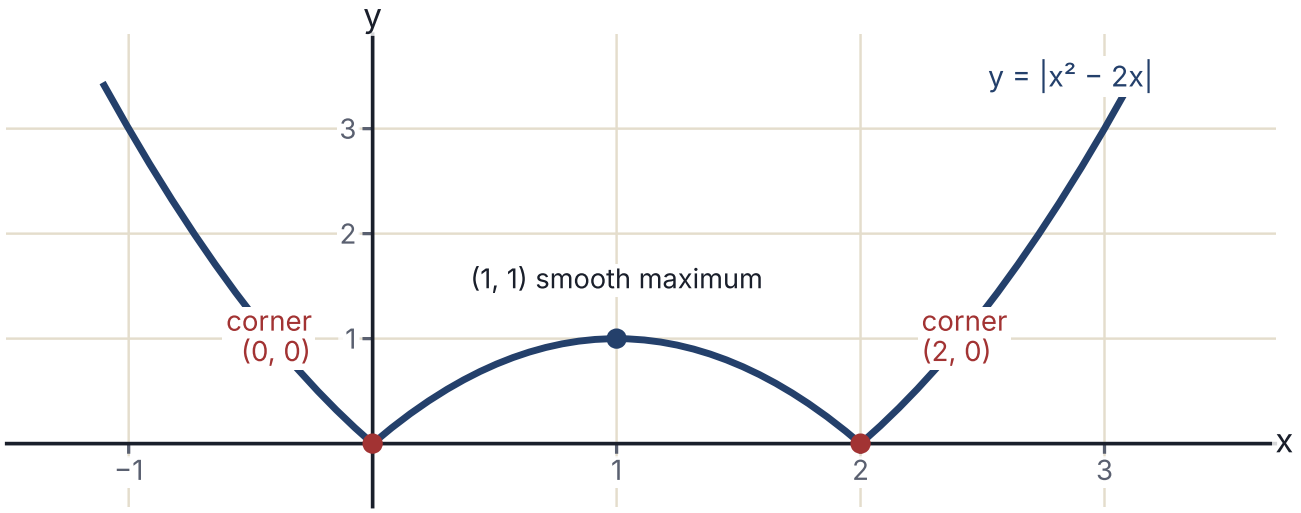
The median m satisfies $F(m) = \dfrac{1}{2}$: $\dfrac{m^2 + m}{6} = \dfrac{1}{2}$, so $m^2 + m - 3 = 0$ and $m = \dfrac{-1 + \sqrt{13}}{2} \approx 1.30$ (the other root is negative, outside $0 \leq m \leq 2$).

(B)

(A) 1 is the midpoint of the interval, the median only of a uniform distribution. (C) $\dfrac{11}{9}$ is the mean: $f(x) = F'(x) = \dfrac{2x + 1}{6}$ and $\int_0^2 x \cdot \dfrac{2x + 1}{6}\, dx = \dfrac{11}{9}$. (D) solves $F(m) = \dfrac{1}{4}$, which gives the lower quartile, not the median.

10

$y = x^2 - 2x$ is negative for $0 < x < 2$, and the absolute value reflects that part up. The graph is continuous everywhere, with sharp corners at $x = 0$ and $x = 2$: at $x = 0$ the left and right gradients are -2 and 2, and at $x = 2$ they are -2 and 2, so the limit defining $f'(x)$ does not exist there.



Question 10 · $y = |x^2 - 2x|$: corners at $x = 0$ and $x = 2$, a smooth maximum at $x = 1$

(A)

(B) and (C) include $x = 1$, but there the graph has a smooth local maximum with a horizontal tangent: $f'(1) = 0$ exists. (D) forgets that the reflection creates corners where the graph meets the x-axis.

Section I key

1 B · 2 B · 3 C · 4 A · 5 D · 6 D · 7 A · 8 C · 9 B · 10 A

For each item you missed, find the distractor you chose above: it names the exact error that produces it. Write that error down before you look at Section II.

Worked solutions · Section II

11

$$\frac{x^2 - 9}{x^2 + x - 6} \div \frac{x - 3}{2x - 4} = \frac{(x-3)(x+3)}{(x+3)(x-2)} \times \frac{2(x-2)}{x-3}$$ factorise, then multiply by the reciprocal

$= 2$ cancel $(x - 3)$, $(x + 3)$ and $(x - 2)$

(2) (for $x \neq 3, -3, 2$)

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Correctly factorises at least two of the expressions, or equivalent merit	1

12

Product rule with $u = x^2$, $v = e^{3x}$, so $u' = 2x$ and $v' = 3e^{3x}$:

$$\frac{dy}{dx} = 2xe^{3x} + 3x^2e^{3x} = xe^{3x}(2 + 3x)$$

$\frac{dy}{dx} = xe^{3x}(3x+2)$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct derivative	2
Correctly differentiates e^{3x}, or applies the product rule with one error, or equivalent merit	1

13

The terms $3k-2$ form an AP: first term ($k=5$) 13, last term ($k=24$) 70, and there are $24-5+1=20$ terms.

$$\sum_{k=5}^{24}(3k-2) = \frac{20}{2}(13+70) = 10 \times 83 = 830$$

830

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Identifies the first term, the last term or the number of terms correctly, or equivalent merit	1

Counting the terms as $24-5=19$ is the usual slip: from $k=5$ to $k=24$ inclusive there are 20 terms.

14 (a)

The numerator is 3 times the derivative of the denominator: $6x = 3 \times 2x$.

$$\int \frac{6x}{x^2+4}\,dx = 3\int \frac{2x}{x^2+4}\,dx = 3\ln\left(x^2+4\right)+C$$

$3\ln\left(x^2+4\right)+C$ (no absolute value needed: $x^2+4>0$)

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct primitive	2
Recognises the form $\frac{f'(x)}{f(x)}$, or obtains $k\ln\left(x^2+4\right)$ with $k \neq 3$, or equivalent merit	1

14 (b)

$\frac{x+2}{\sqrt{x}} = x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}$ split the fraction

$\int_1^4 \left(x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}\right)dx = \left[\frac{2}{3}x^{\frac{3}{2}} + 4x^{\frac{1}{2}}\right]_1^4$

$= \left(\frac{16}{3}+8\right) - \left(\frac{2}{3}+4\right) = \frac{14}{3}+4 = \frac{26}{3}$

$\frac{26}{3}$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Finds a correct primitive, or equivalent merit	2
Writes the integrand as a sum of powers of x, or equivalent merit	1

Dividing each term by $\sqrt{x}$ first is the whole question. Integrating the numerator and the denominator separately is not a rule.

15

$(2\cos x + 1)(\cos x + 1) = 0$ a quadratic in $\cos x$

$\cos x = -1 : \ x = \pi$

$\cos x = -\frac{1}{2} : \ x = \pi - \frac{\pi}{3},\ \pi + \frac{\pi}{3} = \frac{2\pi}{3},\ \frac{4\pi}{3}$ second and third quadrants, related angle $\frac{\pi}{3}$

$x = \dfrac{2\pi}{3}, \pi, \dfrac{4\pi}{3}$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Factorises and solves one of the two equations correctly, or equivalent merit	2
Factorises the quadratic in $\cos x$, or equivalent merit	1

16

$\left(x^2 - 10x + 25\right) + \left(y^2 + 6y + 9\right) = -9 + 25 + 9$ complete both squares

$(x - 5)^2 + (y + 3)^2 = 25$

Centre $(5, -3)$, radius 5.

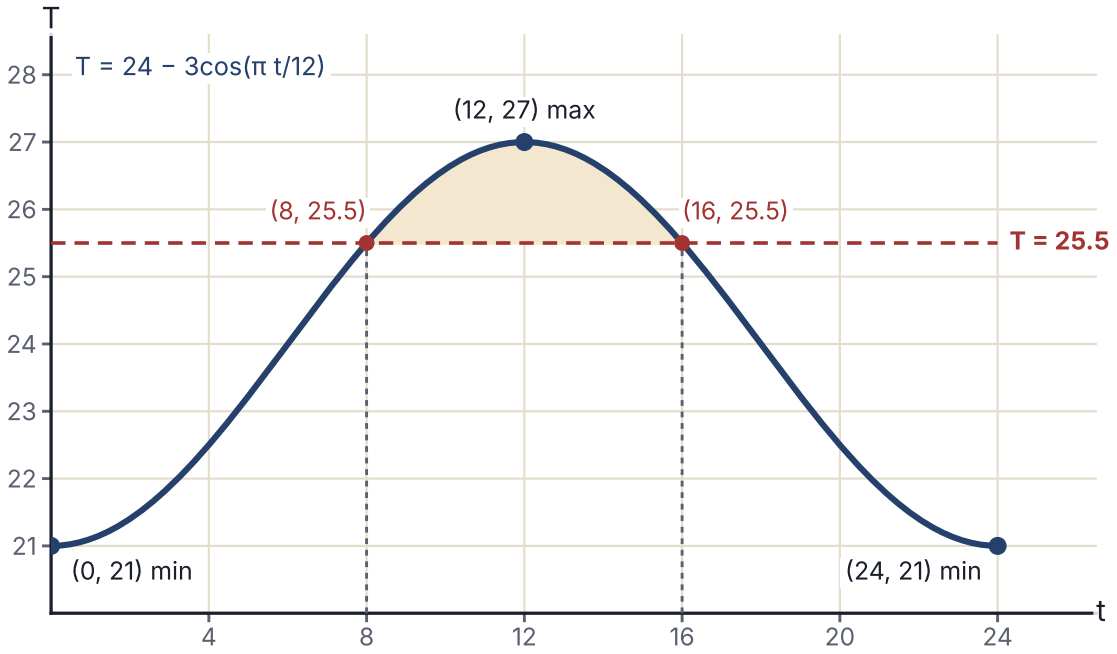
MARKING GUIDELINES

CRITERIA	MARKS
Provides correct centre and radius	2
Completes the square in x or in y correctly, or equivalent merit	1

A frequent error is $(x + 5)^2 + (y - 3)^2$, reading the signs of the centre backwards. Another is forgetting to move the constant 9 to the right-hand side.

17 (a)

The graph of $y = \cos x$ is stretched to period $\dfrac{2\pi}{\pi/12} = 24$ hours and amplitude 3, reflected in its axis by the minus sign, and moved up 24. So T is least, 21, when $\cos \frac{\pi t}{12} = 1$ ($t = 0$ and $t = 24$, midnight) and greatest, 27, when $\cos \frac{\pi t}{12} = -1$ ($t = 12$, noon).



Question 17 · the water temperature, with the heating off where $T \geq 25.5$

Minimum 21°C at midnight ($t = 0$ and $t = 24$); maximum 27°C at noon ($t = 12$)

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct sketch	2
Shows the correct period or the correct maximum and minimum, or equivalent merit	1

17 (b)

$24 - 3\cos\left(\frac{\pi t}{12}\right) = 25.5 \implies \cos\left(\frac{\pi t}{12}\right) = -\frac{1}{2}$ the boundary

$\frac{\pi t}{12} = \frac{2\pi}{3}$ or $\frac{4\pi}{3} \quad \left(0 \leq \frac{\pi t}{12} \leq 2\pi\right)$

$t = 8$ or $t = 16$

From the graph, $T \geq 25.5$ between these times, around the noon maximum.

The heating is off from 8 am to 4 pm ($8 \leq t \leq 16$).

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Finds $t = 8$ and $t = 16$, or equivalent merit	2
Obtains $\cos\left(\frac{\pi t}{12}\right) = -\frac{1}{2}$, or equivalent merit	1

Check the interval against the graph. Solving with the wrong sign, $\cos = +\frac{1}{2}$, gives $t = 4$ and $t = 20$, which are cool times of the day.

18 (a)

$P(W) = P(H)P(W \mid H) + P(A)P(W \mid A) = 0.6 \times 0.7 + 0.4 \times 0.4$ two branches of the tree

$= 0.42 + 0.16 = 0.58$

0.58

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Finds one of 0.42 or 0.16, or equivalent merit	1

18 (b)

$$P(H \mid W) = \frac{P(H \cap W)}{P(W)} = \frac{0.42}{0.58} = \frac{21}{29} \approx 0.724$$

$\frac{21}{29}$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Uses $P(H \cap W) = 0.42$ in a conditional probability, or equivalent merit	1

The answer is not 0.7: that is $P(W \mid H)$, the probability of a win given a home game. Here the condition is the win.

19 (a)

$0.1 + 0.2 + p + 0.3 + 0.1 = 1$, so $p = 0.3$.

$$\mathrm{E}(X) = 0(0.1) + 5(0.2) + 7(0.3) + 9(0.3) + 10(0.1) = 1 + 2.1 + 2.7 + 1 = 6.8$$

$p = 0.3, \mathrm{E}(X) = 6.8$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct p and $\mathrm{E}(X)$	2
Finds p, or equivalent merit	1

19 (b)

$\mathrm{E}\left(X^2\right) = 0 + 25(0.2) + 49(0.3) + 81(0.3) + 100(0.1) = 5 + 14.7 + 24.3 + 10 = 54$

$\mathrm{Var}(X) = \mathrm{E}\left(X^2\right) - \mu^2 = 54 - 6.8^2 = 54 - 46.24 = 7.76$

$\mathrm{Var}(X) = 7.76$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Finds $\mathrm{E}\left(X^2\right) = 54$, or equivalent merit	1

$\mathrm{E}\left(X^2\right)$ squares the values, not the probabilities. Squaring the probabilities is the error that loses both marks.

20

Let the intensities be I_1 (at 88 dB) and I_2 (at 106 dB). Subtract the two equations:

$$106 - 88 = 10\log_{10}\frac{I_2}{I_0} - 10\log_{10}\frac{I_1}{I_0} = 10\log_{10}\frac{I_2}{I_1} \quad \text{log law}$$

$$\log_{10}\frac{I_2}{I_1} = 1.8 \implies \frac{I_2}{I_1} = 10^{1.8} \approx 63.1$$

The louder noise was about 63 times as intense.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Obtains $\log_{10}\frac{I_2}{I_1} = 1.8$, or equivalent merit	1

An increase of 18 dB is not 18 times, nor 1.8 times, as intense. Every 10 dB multiplies the intensity by 10: this is what a logarithmic scale does.

21

At P, the bearing back to S is $060^\circ + 180^\circ = 240^\circ$ and the bearing to Q is 170°, so $\angle SPQ = 240^\circ - 170^\circ = 70^\circ$.

$$SQ^2 = 3.2^2 + 4.5^2 - 2(3.2)(4.5)\cos 70^\circ \approx 20.640 \quad \text{cosine rule}$$

$$SQ \approx 4.54 \text{ km}$$

$$\sin\angle PQS = \frac{3.2\sin 70^\circ}{SQ} \approx 0.6619 \implies \angle PQS \approx 41.4^\circ \quad \text{sine rule; acute, as } SP \text{ is not the longest side}$$

$$\text{bearing of } P \text{ from } Q = 170^\circ + 180^\circ = 350^\circ$$

$$\text{bearing of } S \text{ from } Q = 350^\circ - 41.4^\circ \approx 308.6^\circ \quad S \text{ lies anticlockwise from } QP$$

$SQ \approx 4.54$km; the bearing of S from Q is about 309°

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	4
Finds SQ and $\angle PQS$, or equivalent merit	3
Finds SQ, or finds $\angle SPQ = 70°$ and applies the cosine rule with one error, or equivalent merit	2
Finds $\angle SPQ = 70°$, or equivalent merit	1

Draw the north line at P and mark both bearings on it before finding any angle of the triangle. Using $170° - 60° = 110°$ as the angle at P is the most common error.

22

$\ln(2x-1) = 0 \implies 2x - 1 = 1 \implies x = 1$ the x-intercept $(1, 0)$

$\dfrac{dy}{dx} = \dfrac{2}{2x-1} = 2$ at $x = 1$ chain rule

$y - 0 = 2(x - 1)$ point-gradient form

$y = 2x - 2$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Finds the point $(1, 0)$ and the gradient 2, or equivalent merit	2
Finds $\dfrac{dy}{dx} = \dfrac{2}{2x-1}$ or the point $(1, 0)$, or equivalent merit	1

23 (a)

$a = \dfrac{dv}{dt} = 12 \times 0.8e^{-0.8t} = 9.6e^{-0.8t}$, so at $t = 0$, $a = 9.6$ m/s^2.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct answer	1

23 (b)

The velocity is positive for $t > 0$, so the distance is the displacement from the start.

$x = \int 12\left(1 - e^{-0.8t}\right) dt = 12t + 15e^{-0.8t} + C \quad \dfrac{12}{0.8} = 15$

$x = 0$ at $t = 0: \ 0 = 15 + C, \ C = -15$

$x(10) = 120 + 15e^{-8} - 15 = 105 + 15e^{-8} \approx 105.005$

The sprinter runs about 105 metres.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Finds $x = 12t + 15e^{-0.8t} - 15$, or equivalent merit	2
Integrates v to obtain $12t + 15e^{-0.8t}$, or equivalent merit	1

Without the constant, $x(10) = 120 + 15e^{-8} \approx 120$ m, which is too far. Equivalently, evaluate $\int_0^{10} v\,dt$, where the lower limit takes care of the constant.

24 (a)

The weekly distances form a GP with $a = 60$ and $r = 1.1$, so in week n she rides $60(1.1)^{n-1}$ km.

$$60(1.1)^{n-1} > 150 \implies 1.1^{n-1} > 2.5$$

$$n - 1 > \frac{\ln 2.5}{\ln 1.1} \approx 9.61 \implies n > 10.61 \quad \text{take logarithms}$$

Check: week 10 gives $60(1.1)^9 \approx 141.5$ km and week 11 gives $60(1.1)^{10} \approx 155.6$ km.

Week 11

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Forms the inequality $60(1.1)^{n-1} > 150$, or equivalent merit	1

24 (b)

$$S_{11} = \frac{60\left(1.1^{11} - 1\right)}{1.1 - 1} = 600\left(1.1^{11} - 1\right) \approx 1111.87$$

1112 km

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct answer	1

25 (a)

$$\int_0^3 \left(x^2 - 2x\right) dx = \left[\frac{x^3}{3} - x^2\right]_0^3 = (9 - 9) - 0 = 0$$

0

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct answer	1

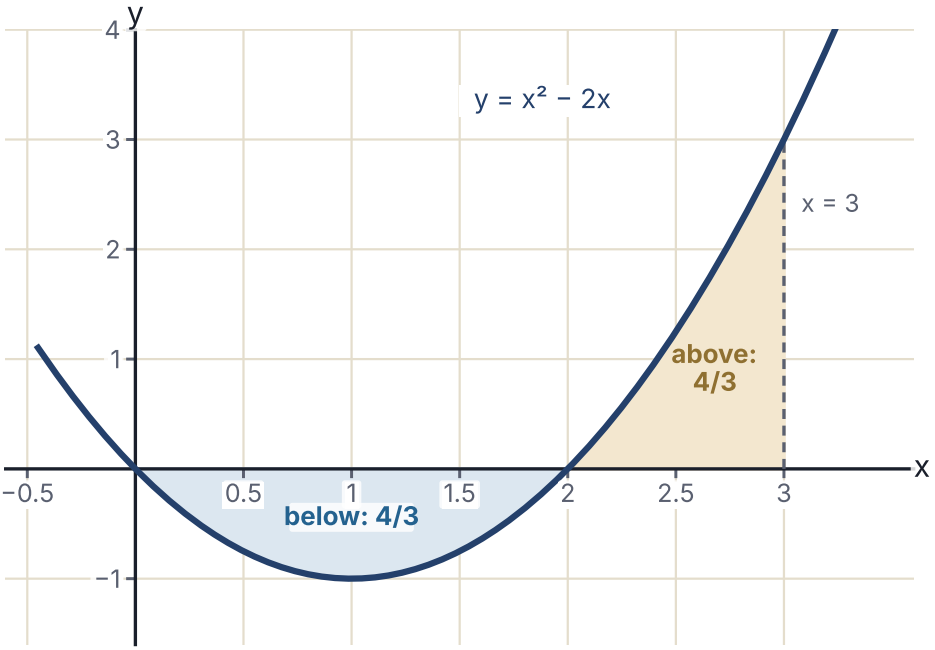
25 (b)

$y = x(x-2)$ is below the x-axis for $0 < x < 2$ and above it for $2 < x < 3$, so the two parts are found separately.

$$\int_0^2 \left(x^2 - 2x\right) dx = \tfrac{8}{3} - 4 = -\tfrac{4}{3} \quad \text{below the axis}$$

$$\int_2^3 \left(x^2 - 2x\right) dx = 0 - \left(-\tfrac{4}{3}\right) = \tfrac{4}{3} \quad \text{above the axis}$$

$$\text{Area} = \left|-\tfrac{4}{3}\right| + \tfrac{4}{3} = \tfrac{8}{3}$$



Question 25 · equal areas below and above the axis, so the integral in (a) is 0

$\frac{8}{3}$ square units

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Recognises that the region below the axis must be treated separately, or finds one of the two areas, or equivalent merit	1

Part (a) is the trap set for part (b): a definite integral counts area below the axis as negative, so an integral of 0 does not mean an area of 0.

26

Distance is the area under the velocity graph. With $h = 10$:

$$\int_0^{40} v\,dt \approx \frac{10}{2}\left[0 + 4.6 + 2(3.2 + 4.1 + 4.5)\right] = 5 \times 28.2 = 141$$

The boat travels about 141 m.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Applies the trapezoidal rule with one error, or equivalent merit	1

27

$$\log_2 [x(x+2)] = 3 \implies x(x+2) = 2^3 = 8 \quad \text{log law}$$

$$x^2 + 2x - 8 = 0 \implies (x+4)(x-2) = 0$$

$$x = 2 \quad (x = -4 \text{ is rejected: } \log_2 x \text{ needs } x > 0)$$

$x = 2$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Obtains $x(x+2) = 8$, or equivalent merit	1

28 (a)

The deposit made at the end of year k earns interest for $n - k$ years. Immediately after the nth deposit:

$$A_n = 5000 + 5000(1.04) + 5000(1.04)^2 + \cdots + 5000(1.04)^{n-1} \quad \text{a GP: } a = 5000,\ r = 1.04$$

$$= \frac{5000\,(1.04^n - 1)}{1.04 - 1} = 125\,000\,(1.04^n - 1)$$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct proof	2
Writes the amount as a geometric series, or equivalent merit	1

28 (b)

$$125\,000\,(1.04^n - 1) > 80\,000 \implies 1.04^n > 1.64$$

$$n > \frac{\ln 1.64}{\ln 1.04} \approx 12.6$$

Check: $A_{12} \approx \$75\,129$ and $A_{13} \approx \$83\,134$.

After the 13th deposit

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Obtains $1.04^n > 1.64$, or equivalent merit	1

28 (c)

With an annual deposit of $\$D$, the series of part (a) gives $\dfrac{D\left(1.04^{10}-1\right)}{0.04}$ after 10 deposits.

$$\frac{D\left(1.04^{10}-1\right)}{0.04} = 80\,000 \implies D = \frac{3200}{1.04^{10}-1} \approx 6663.28$$

$\$6663.28$ each year

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Forms a correct equation for the deposit, or equivalent merit	1

Keep 1.04^{10} in the calculator: rounding it to 1.48 gives $\$6666.67$, which misses the nearest cent.

29 (a)

$$f'(x) = 4x^3 + 6x^2 = 2x^2(2x+3)$$

$$f'(x) = 0 \implies x = 0 \text{ or } x = -\tfrac{3}{2}; \quad f(0) = 0,\ f\left(-\tfrac{3}{2}\right) = \tfrac{81}{16} - \tfrac{27}{4} = -\tfrac{27}{16}$$

x	-2	$-\frac{3}{2}$	-1	0	1
$f'(x)$	-8	0	2	0	10
slope	falling	flat	rising	flat	rising

At $x = -\frac{3}{2}$ the gradient changes from negative to positive: a local minimum. At $x = 0$ it is positive on both sides: a horizontal point of inflection.

Local minimum at $\left(-\frac{3}{2}, -\frac{27}{16}\right)$; horizontal point of inflection at $(0, 0)$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Finds both stationary points and correctly determines the nature of one, or equivalent merit	2
Finds $f'(x)$ and solves $f'(x) = 0$, or equivalent merit	1

Do not call $(0, 0)$ a turning point: $f'(x)$ does not change sign there. The second derivative test fails at $x = 0$, because $f''(0) = 0$; use the table.

29 (b)

$$f''(x) = 12x^2 + 12x = 12x(x+1) = 0 \implies x = -1 \text{ or } x = 0$$

x	-2	-1	$-\frac{1}{2}$	0	1
$f''(x)$	24	0	-3	0	24
concavity	up		down		up

The concavity changes at both values, and $f(-1) = 1 - 2 = -1$.

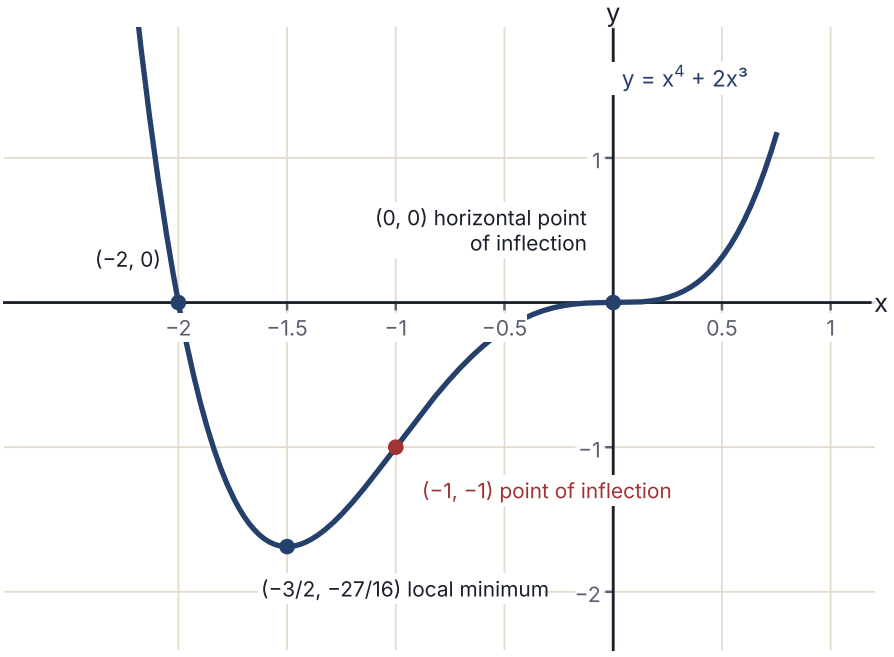
Points of inflection at $(-1, -1)$ and $(0, 0)$ (horizontal)

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Finds $x = -1$ and $x = 0$ from $f''(x) = 0$, or equivalent merit	1

29 (c)

$f(x) = x^3(x+2)$, so the curve meets the x-axis at $(-2, 0)$ and $(0, 0)$. As $x \to \pm\infty$, $f(x) \to \infty$.



Question 29 · $y = x^4 + 2x^3$

Sketch as shown

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct sketch showing the stationary points, the points of inflection and the intercepts	2
Provides a sketch with the correct shape, or equivalent merit	1

30

$\frac{dy}{dx} = 3x^2 - 6x = 9$ parallel lines have equal gradients

$x^2 - 2x - 3 = 0 \implies (x-3)(x+1) = 0$

$x = 3:\ y = 27 - 27 + 1 = 1; \quad x = -1:\ y = -1 - 3 + 1 = -3$

$(3, 1)$ and $(-1, -3)$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	3
Finds $x = 3$ and $x = -1$, or equivalent merit	2
Sets $\frac{dy}{dx} = 9$, or equivalent merit	1

31 (a)

$\int_0^5 k\left(5t - t^2\right) dt = k\left[\frac{5t^2}{2} - \frac{t^3}{3}\right]_0^5 = k\left(\frac{125}{2} - \frac{125}{3}\right) = \frac{125k}{6}$

$\frac{125k}{6} = 1 \implies k = \frac{6}{125}$ total probability 1

Also $5t - t^2 = t(5-t) \geq 0$ on $0 \leq t \leq 5$ so $f(t) \geq 0$ $k = \frac{6}{125}$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct proof	1

31 (b)

$$P(T \leq 1) = \frac{6}{125}\left[\frac{5t^2}{2} - \frac{t^3}{3}\right]_0^1 = \frac{6}{125}\left(\frac{5}{2} - \frac{1}{3}\right) = \frac{6}{125} \times \frac{13}{6} = \frac{13}{125}$$

$\frac{13}{125}$ (that is, 0.104)

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Writes $P(T \leq 1)$ as a correct integral, or equivalent merit	1

32

$$\int_1^k (2x-1)\,dx = \left[x^2 - x\right]_1^k = k^2 - k - 0 = 6$$

$$k^2 - k - 6 = 0 \implies (k-3)(k+2) = 0$$

Since $k > 1$, $\boxed{k = 3}$.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Obtains $k^2 - k = 6$, or equivalent merit	1

33

$$\frac{dy}{dx} = 2e^{2x}\cos x - e^{2x}\sin x = e^{2x}(2\cos x - \sin x) \quad \text{product rule}$$

$$\frac{d^2y}{dx^2} = 2e^{2x}(2\cos x - \sin x) + e^{2x}(-2\sin x - \cos x) = e^{2x}(3\cos x - 4\sin x) \quad \text{product rule again}$$

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = e^{2x}\left[(3\cos x - 4\sin x) - (8\cos x - 4\sin x) + 5\cos x\right]$$

$$= e^{2x}\left[0\cos x + 0\sin x\right] = 0$$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct proof	3
Finds $\dfrac{d^2y}{dx^2}$ correctly, or equivalent merit	2
Finds $\dfrac{dy}{dx}$ correctly, or equivalent merit	1

Factor out e^{2x} at every step. It keeps the second product rule short and makes the cancellation in the last line easy to see.

34 (a)

$24 = 28 - 4$ is one standard deviation below the mean and $36 = 28 + 2 \times 4$ is two above. By symmetry, half of 68% lies between $\mu - \sigma$ and μ, and half of 95% between μ and $\mu + 2\sigma$:

$$34\% + 47.5\% = 81.5\%$$

$\boxed{81.5\%}$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Finds 34% or 47.5%, or equivalent merit	1

34 (b)

$$z_{\text{run}} = \frac{25 - 28}{4} = -0.75, \qquad z_{\text{swim}} = \frac{52 - 56}{2.5} = -1.6$$

In both events a lower time is better. Her swim was 1.6 standard deviations below the mean, further below than her run (0.75). Mia performed better in the swimming race.

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct answer with justification	2
Finds both z-scores, or equivalent merit	1

With times, the more negative z-score is the better performance. Choosing the run because $-0.75 > -1.6$ treats the times as if a higher time were better.

35

$$f(x) = \int 2\cos 2x \, dx = \sin 2x + C$$

$$3 = \sin \tfrac{\pi}{2} + C = 1 + C \implies C = 2$$

$y = \sin 2x + 2$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	2
Finds the primitive $\sin 2x + C$, or equivalent merit	1

36

The series is geometric with $a = 1$ and $r = \dfrac{x-1}{2}$.

$$S_\infty = \frac{1}{1 - \frac{x-1}{2}} = \frac{2}{3 - x} = 4$$

$$3 - x = \tfrac{1}{2} \implies x = \tfrac{5}{2}$$

Then $r = \dfrac{\frac{5}{2} - 1}{2} = \dfrac{3}{4}$, and $|r| < 1$, so the limiting sum exists. $x = \dfrac{5}{2}$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution with justification	3
Finds $x = \dfrac{5}{2}$, or equivalent merit	2
Writes S_∞ in terms of x, or states $\left\lvert \dfrac{x-1}{2} \right\rvert < 1$, or equivalent merit	1

The formula $S_\infty = \frac{a}{1-r}$ is only true when $|r| < 1$. Solving the equation is not enough: the last mark is for checking the ratio.

37 (a)

The inside edge is $2x + 2\pi r = 400$, so $x = 200 - \pi r$. The field is x m by $2r$ m:

$$A = 2rx = 2r(200 - \pi r) = 400r - 2\pi r^2$$

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct proof	1

37 (b)

Both $r > 0$ and $x = 200 - \pi r > 0$, so $0 < r < \dfrac{200}{\pi}$.

$$\frac{dA}{dr} = 400 - 4\pi r = 0 \implies r = \frac{100}{\pi} \approx 31.83$$

$$\frac{d^2A}{dr^2} = -4\pi < 0 \quad \text{concave down: a maximum}$$

$$x = 200 - \pi \times \frac{100}{\pi} = 100, \qquad 2r = \frac{200}{\pi} \approx 63.66$$

$$A = 2 \times \frac{100}{\pi} \times 100 = \frac{20\,000}{\pi} \approx 6366.2$$

A is a quadratic in r that is concave down, so the single stationary point is the greatest area on the domain.

The field is 100 m by $\dfrac{200}{\pi} \approx 63.7$ m, with greatest area about 6366 m^2

MARKING GUIDELINES

CRITERIA	MARKS
Provides correct solution	4
Finds $r = \dfrac{100}{\pi}$ with justification of the maximum and finds the dimensions or the area, or equivalent merit	3
Finds $r = \dfrac{100}{\pi}$, or equivalent merit	2
Finds $\dfrac{dA}{dr} = 400 - 4\pi r$, or equivalent merit	1

The dimensions asked for are those of the field, x and $2r$, not r alone. Half of the edge is straight and half is curved at the maximum: $2x = 2\pi r = 200$.

What this paper examined

Focus area	Section I	Section II	Marks	Target
Year 11 course (assumed knowledge)	2	20	**22**	22
Further graph transformations and modelling	1	7	**8**	8
Sequences and series	1	8	**9**	9
Differential calculus	1	8	**9**	9
Integral calculus	1	14	**15**	15
Applications of calculus	1	16	**17**	17
Random variables	2	11	**13**	13
Financial mathematics	1	6	**7**	7
Total	**10**	**90**	**100**	**100**

MA99 aims for about a fifth of the marks to come from the Year 11 course (by MA99's count, NESA's sample paper has 24), with the rest spread over the seven Year 12 focus areas in proportion to their size. Where each was examined:

- **Year 11 course:** Questions 1 and 6; 11 algebraic fractions; 15 a trigonometric equation; 16 a circle; 18 conditional probability; 21 bearings with the sine and cosine rules; 27 a logarithmic equation; 30 tangents of a given gradient.
- **Further graph transformations and modelling (MAV-12-01, MAV-12-02):** Question 7; 17 a periodic model; 20 the decibel scale.
- **Sequences and series (MAV-12-03):** Question 2; 13 summation notation; 24 a geometric growth model; 36 a limiting sum.
- **Differential calculus (MAV-12-04):** Question 3; 12 the product rule; 22 a tangent to a logarithmic curve; 33 a second derivative identity.
- **Integral calculus (MAV-12-05):** Question 4; 14 indefinite and definite integrals; 25 a signed area; 26 the trapezoidal rule; 32 a definite integral equation; 35 a curve from its gradient function.
- **Applications of calculus (MAV-12-06):** Question 10; 23 motion from a velocity; 29 stationary points, inflections and a sketch; 37 optimisation.
- **Random variables (MAV-12-07):** Questions 5 and 9; 19 expectation and variance; 31 a probability density function; 34 the normal distribution and z-scores.
- **Financial mathematics (MAV-12-08):** Question 8; 28 an annuity.

The questions rise in difficulty through the paper. Use the table to find the focus area where you lost the most marks, and work through that focus area's chapter of the Year 12 Complete Textbook before you sit Practice Paper 2.

End of worked solutions.

About this resource

MA99 EDUCATION

Scope

NSW Mathematics Advanced, Year 12. Written to the NSW Mathematics Advanced 11-12 Syllabus (2024), an HSC style paper on the whole course: the Year 12 focus areas, with the Year 11 course as assumed knowledge, for students sitting the HSC in 2027 and later years. Every value and solution was re-derived independently by computer algebra before printing. Contexts in the questions are invented unless a source is named.

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